VIBRATION ANALYSIS OF CABLE NETWORKS AND BEAM GRILLAGE BY FINITE ELEMENT - TRANSFER MATRIX METHOD

by

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CERTIFICATE

27/2/87

This is to certify that the thesis entitled, "VIBRATION ANALYSIS OF CABLE NETWORKS AND BEAM GRILLAGE BY FINITE ELEMENT-TRANSFER MATRIX METHOD" is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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NOMENCLATURE

B	Suffix used for bottom boundary
c	Wave equation constant
C	Torsional rigidity
{D}	Displacement vector
E	Modulus of elasticity
E_x, E_y	Modulus of elasticity of elements lying in X direction and Y direction respectively
e	Superscript used for elements
{F}	Force vector
F	Nondimensional force
f	Transverse loading
g	Suffix used for global coordinate
G	Shear modulus
G_x, G_y	Shear modulus for elements lying in X direction and Y direction respectively
I	Area moment of inertia
I_x, I_y	Area moment of inertia of the beam element lying in X direction and Y direction respectively
i	Suffix used for X direction
I_p	Polar moment of inertia of rectangular section
I_{px}, I_{py}	Polar moment of inertia of rectangular section of beam lying in X direction and Y direction respectively
I_{pc}	Polar moment of inertia of equivalent circular section
I_{pcx}, I_{pcy}	Polar moment of inertia of equivalent circular section of beam lying in X direction and Y direction respectively
j	Suffix used for Y direction
[K]	Stiffness matrix
l	Length of cable element

l_x, l_y	Length of cable element lying in X direction and Y direction respectively
L	Suffix used for left hand side
m	Mass per unit length; number of cables lying in X directio
M	Bending moment
M_x, M_y	Bending moments in X direction and Y direction beams respectively
$\overline{M}_x$, $\overline{M}_y$	Non dimensional bending moments in X direction and Y direction beams respectively
n	Number of cables lying in Y direction
p, p_1, p_2, p_3	Non dimensional frequency
r_m	Mass ratio
r_T	Tension ratio
R	Suffix used for right hand side
S	Stations
t	Time
T	Tension; torque; suffix for top boundary
$\overline{T}_x$, $\overline{T}_y$	Non dimensional torque in X direction and Y direction beams respectively
$[T]$	Transfer matrix
$[U]$	Combined transfer matrix relating state vectors at boundaries
V	Shear force
$\overline{V}_x$, $\overline{V}_y$	Non dimensional shear forces for X direction and Y direction beams respectively
W	Transverse displacement
$\overline{W}_x$, $\overline{W}_y$	Non dimensional displacements for X direction and Y direction beams respectively
X, Y, Z	Coordinate axes
$\{Z\}$	State vector
θ	Angle of twist

$\bar{\theta}_x$, $\bar{\theta}_y$	Non dimensional angles of twist for X direction and Y direction beams respectively
η	Length ratio
ω	Circular frequency of harmonic motion
$\{\eta\}$	Non dimensional displacement vector for cables
β	Non dimensional frequency
Ψ	Slope
$\bar{\Psi}_x$, $\bar{\Psi}_y$	Non dimensional slope of X direction and Y direction beams respectively
ρ	Density

ABSTRACT

In the present work, the combined finite element-transfer matrix (FETM) method has been developed for the vibration analysis of cable networks and beam grillage. Firstly, the system is divided into elements and their transfer matrices are obtained from the closed form solutions of their equations of motion. From these transfer matrices, stiffness matrices are developed to obtain the transfer matrix for the repeated section using finite element method (FEM). This transfer matrix is then used to derive the transfer matrix for the whole system. Thereafter, the boundary conditions are applied to obtain the characteristic equation, roots of which give the natural frequencies of the system.

For cable networks, the transverse vibrations of the cables are considered. Cases of the cable networks, orthogonal and non-orthogonal are dealt with. Transverse vibrations of the beam grillage in bending and combined bending-torsional modes with various types of boundary conditions are analysed.

It is conjectured that FETM utilizes lower memory and has a simpler formulation than FEM for the class of repeated system. It has the same advantage of obtaining formulation from the exact solution of its elements, as is for transfer matrix method.

CHAPTER 1

INTRODUCTION

The finite element method (FEM) is the most powerful tool hence widely used for the vibration analysis of structures. However, for complex structure, the method requires a large number of nodes resulting in a very large computer memory requirement.

Dokainish [1] used the combined finite element-transfer matrix method (FETM) in the study of the dynamics of rectangular plates. In his method, the size of stiffness and mass matrices, is equal to the number of degrees of freedom of one strip, so the frequency determinant for a clamped-clamped plate is 18 x 18 by the FETM as compared to a 108 x 108 matrix eigen value problem obtained by using the standard FEM with same number of nodes. McDaniel and Eversole [2] have proposed a similar approach in treating a stiffened plate structure. Sankar and Hoa [3] offer ~~after~~ an approach, in which an extended transfer matrix relating the state vectors which consists of state variables (displacements and forces) and their derivatives with respect to frequency was used. Ohga and Shigematsu [4] have also used FETM for bending and buckling problems and have developed a technique for treating the structures with intermediate conditions.

The cable networks are used for covering large column-free areas. This is done due to their light weight and high strength resulting in reduced cost per unit covered area. It

also adds to aesthetic sense. The cables have negligible flexural rigidity and serve primarily as the tension members. Considerable work has been done in studying cable networks subjected to static loading whereas relatively a little work has been done in the study of dynamic response of cable networks [5] more so due to the complexity of the shape. The static and dynamic analyses of the cable networks have been done by modelling as discrete systems. This approach leads to a set of simultaneous algebraic and differential equations respectively and it becomes quite involved in terms of computer time and memory as the number of equilibrium equations are generally large. The memory requirement also become very large if consistent mass-matrix approach is used in the dynamic analysis.

The finite element method with lumped mass approach has been used by Choudhari [6]. Singh [7] has used the FEM with the consistent mass-matrix approach. In his work, a cable between two joints has been taken as one finite element for the analysis of the cable networks.

A grillage/grid is a structure consisting of two sets of orthogonal beams rigidly connected at the intersections. Some of its applications are in building floors (concrete slabs embedded in grillage), ship decks, bridge floors, missile ground facilities, electronically steerable radar systems etc. Several studies [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18] have been reported of the static and dynamic analyses of the beam grillage. One of them, namely the discrete approach is suitable for the analysis of such systems but is laborious and time consuming.

Furthermore if one uses consistent mass-matrix approach, it can be solved for higher order grillage utilizing large memory computers.

The earlier work on the vibration of grillages is known to be that of Cox and Denke [8] who formulated the problem in the matrix form, suitable for the use of digital computer. Ellington and McCallion [9] have taken into account the discrete nature of the beams, by (a) assuming the mass of beams as concentrated at the intersections (thereby reducing the problem to a finite-degree of freedom system), and (b) neglecting the torsional stiffness of the beams. A more general solution was presented by Thein Wah [10] who extended the method of Ellington and McCallion [9] by taking distributed mass of the beams, instead of lumping the mass at the joints. Such a study is necessary to assess the accuracy of the approximate solutions obtained by lumping the masses. However, due to complexity of the formulation, the solutions obtained are for bending deformation only. Thein Wah's [10] work was extended by Chang and Michelson [11] in which the complicated equations were simplified with the application of Rayleigh-Ritz method.

Several studies have been made in which the grid is replaced by an equivalent orthotropic plate. For its static analysis, Timoshenko [12] considered the grid and gave the parameters of an equivalent orthotropic plate in terms of the known parameters of the grid. Using Timoshenko's results, Taraporewalla [13] found static deflection of the grillage by considering the plate analogy. Renton [14] gave the theoretical basis for the relations given by Timoshenko. He took the advantage of

the resulting regularity of the grids, consisting of a large number of beams. He used the combination of "finite differences", and differential calculus, the two being related by Taylor's expansion. This technique gives differential equation for the deflection of an equivalent continuum. In this further work, Renton [15] gave first and second order continuum approximation for various forms of space grids. In the paper of Cheng [16], the results for the frequency parameters for a 3 x 3 square uniform grid are given, which have been obtained from equivalent orthotropic plate [17]. The results compare well with the results obtained from various discrete methods. Sharma [18] has done the dynamic analysis of simply supported beam grillage based on rational proposition of an orthotropic plate analogy, formulated on the basis of energy equivalence, thus replacing the Timoshenko's plate analogy which is an intuitive proposition. Sharma [18] has further extended the proposition to the case of grillage with clamped edges but the results are limited to the fundamental frequency only.

In the present work, FETM method is used for the free vibration analysis of cable networks and orthogonal beam grillage. Newton-Raphson modified technique is used to determine its natural frequencies.

In Chapter 2, the free vibration analysis for the cable networks using FETM method is given. The problem formulation is done for the cables lying in two directions, orthogonal and non-orthogonal. Chapter 3 deals with the free vibration analysis of clamped orthogonal beam grillage in (a) transverse bending

mode only and (b) combined bending-torsional mode. Method of solution is discussed in Chapter 4. Results for various cable networks and beam grillage are presented in Chapter 4 and compared with earlier results, wherever possible. The conclusions derived from the present work are given in Chapter 5.

CHAPTER 2

VIBRATION ANALYSIS OF CABLE NETWORKS

The geometry of a cable network with rectangular or square boundary can be considered to be consisting of repeated sections, Figure 1c, except the cables in the neighbourhood of the boundaries and hence FETM method is quite suitable for its dynamic analysis. To deal with the transverse vibrations of the cable network with FETM one needs the transfer matrix (TM) for a single cable element which in turn requires its equation of motion and its solution. In the succeeding section, the TM for a single cable is obtained from its equation of motion. From the TM of a cable element, stiffness matrix for the cable element is obtained. Then elemental matrices and state vectors are assembled to obtain the global stiffness matrix and state vector for a repeated section. From the global stiffness matrix, TM relating state vectors on right and left sides of the repeated section is obtained. Because of its repeated characteristic, the system TM is obtained by multiplying the TM^{s} corresponding to number of repeated sections and finally with TM of the left and right hand boundaries. Applying the boundary conditions of the system the characteristic equation is arrived at, which is then solved for its roots. In the present section cables lying in two directions are considered. Directions for networks with cables in two directions are denoted by X and Y.

2.1 TRANSFER MATRIX FOR A CABLE ELEMENT

For a cable shown in Figure 1 with mass per unit length m(X), tension T(X) and transverse loading f(X) per unit length, the equation of motion is [22]

$$\frac{\partial}{\partial X}\left[T(X)\,\frac{\partial W(X,t)}{\partial X}\right] + f(X,t) = m(X)\,\frac{\partial^2 W(X,t)}{\partial t^2} \qquad (2.1)$$

where W is transverse displacement of the cable.

Consider the cable element shown in Figure 1. The TM for the cable is given by equation [19]

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix}_i = \begin{bmatrix} \cos p & \frac{1}{p}\sin p \\ -p\sin p & \cos p \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix}_{i-1} \qquad (2.2a)$$

or $\qquad \{Z\}_i = [T]\,\{Z\}_{i-1}$

where

$$\eta = W/l, \quad p = \omega l/c, \quad F = f_v/T, \quad c = (T/m)^{1/2} \qquad (2.3)$$

l = length of the cable

f_v = vertical component of the force acting on the cable

c = wave equation constant

ω = circular frequency of harmonic motion

and

$\{Z\}_i$ and $\{Z\}_{i-1}$ are state vectors consisting of displacement and force at the station i and i-1 respectively and [T] is TM.

2.1.1 TRANSFER MATRIX FOR THE CABLE LYING IN X DIRECTION

Let l_x, T_x and m_x be the length, tension and mass per unit length respectively for the cable lying in X direction. Then its TM relation is given by

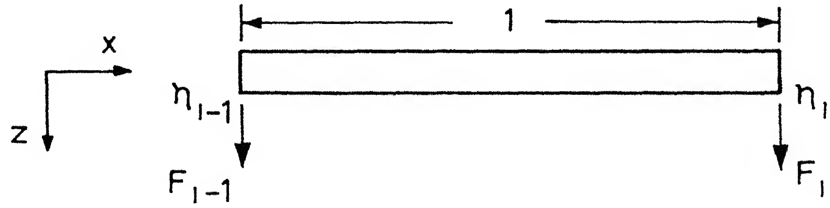


Fig. 1 A cable element
(Non-dimensional quantities)

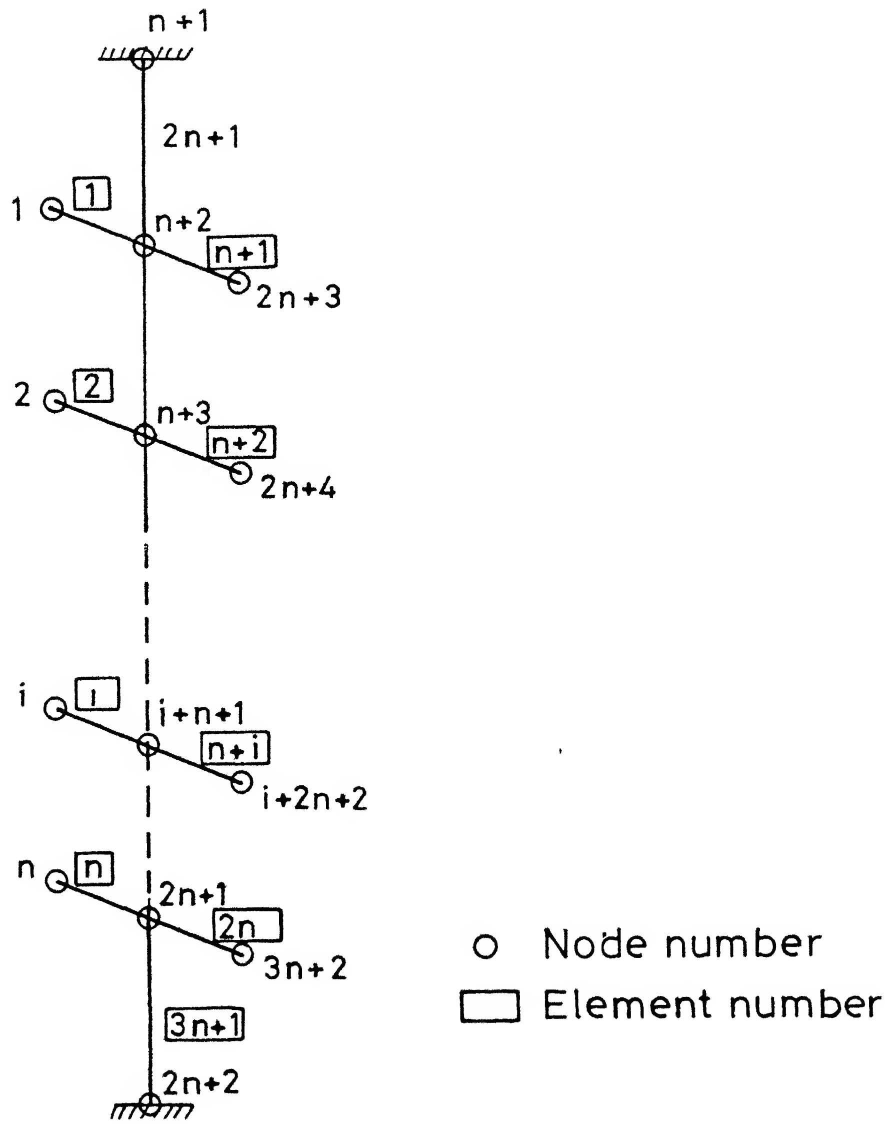


Fig. 2 Finite element mesh of a repeated section.

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix}_i = \begin{bmatrix} \cos p & \frac{1}{p}\sin p \\ -p\sin p & \cos p \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix}_{i-1} \qquad (2.4a)$$

or $\quad \{Z\}_i = [T_1]\,\{Z\}_{i-1} \qquad (2.4b)$

where $\quad \eta = W/l_x, \quad F = f_{vx}/T_x, \quad p = \omega l_x/c \qquad (2.5)$

Note that the state vector of any cable lying in X direction is identified by subscript i.

2.1.2 TRANSFER MATRIX FOR THE CABLE LYING IN Y DIRECTION

Let l_y, T_y and m_y be the length, tension and mass per unit length respectively for the cable lying in the Y direction. Then the TM relation is given by

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix}_j = \begin{bmatrix} \cos p_1\eta_y & \frac{r_T}{p_1}\sin p_1\eta_y \\ -\frac{p_1}{r_T}\sin p_1\eta_y & \cos p_1\eta_y \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix}_{j-1} \qquad (2.6a)$$

or $\quad \{Z\}_j = [T_2]\,\{Z\}_{j-1} \qquad (2.6b)$

where $\quad \eta = W/l_x, \quad p_1^2 = (r_T/r_m)p^2, \quad r_T = T_x/T_y$

$\quad r_m = m_x/m_y, \quad \eta_y = l_y/l_x \qquad (2.7)$

The state vector of any cable lying in the Y direction is identified by subscript j.

2.1.3 TRANSFER MATRIX FOR A REPEATED SECTION OF CABLE NETWORK

To derive the TM relating the left and right hand state vectors (displacement and force) of a repeated section Figure 2,

First it is required to determine the stiffness matrix for the cable element. To do so, equations (2.[illegible]a) and (2.6a) can be transformed into

$$\begin{Bmatrix} -F_{i-1} \\ F_i \end{Bmatrix} = \begin{bmatrix} p\,\dfrac{\cos p}{\sin p} & -\dfrac{p}{\sin p} \\ -\dfrac{p}{\sin p} & p\,\dfrac{\cos p}{\sin p} \end{bmatrix} \begin{Bmatrix} \eta_{i-1} \\ \eta_i \end{Bmatrix} \qquad (2.8a)$$

or $$\{F\}_x = [K_1]\,\{\eta\}_x \qquad (2.8b)$$

where $[K_1]$ is the stiffness matrix of the cable element in X direction.

Similarly for the cable element in Y direction

$$\begin{Bmatrix} -F_{j-1} \\ F_j \end{Bmatrix} = \begin{bmatrix} \dfrac{p_1}{r_T}\,\dfrac{\cos p_1 \eta_1}{\sin p_1 \eta_1} & -\dfrac{p_1}{r_T}\,\dfrac{1}{\sin p_1 \eta_1} \\ -\dfrac{p_1}{r_T}\,\dfrac{1}{\sin p_1 \eta_1} & \dfrac{p_1}{r_T}\,\dfrac{\cos p_1 \eta_1}{\sin p_1 \eta_1} \end{bmatrix} \begin{Bmatrix} \eta_{j-1} \\ \eta_j \end{Bmatrix} \qquad (2.9a)$$

or $$\{F\}_y = [K_2]\,\{\eta\}_y \qquad (2.9b)$$

where $[K_2]$ is the stiffness matrix for the cable element in Y direction.

To develop the TM for the repeated section, shown in Figure 2 from those of its stiffness matrix, a numbering system has been adopted similar to that of FEM. The global stiffness matrix is obtained from those of elements in X and Y directions on the lines of FEM. So there will be a total of 3n + 2 nodes and 3n + 1 elements for n cables lying in the X direction.

The procedure of assembling the element matrices and state vectors is based on the requirement of "compatibility" at

the element nodes. This means that at the nodes where elements are connected, the value(s) of the unknown nodal degree(s) of freedom or variable(s) is(are) the same for all the elements joining at the node. In the present problem, the generalised displacement is η (non-dimensional displacement). When the generalised displacements are matched at a common node, the nodal stiffness and forces of the elements meeting at the node are added to obtain net stiffness and net forces at that node. In view of the above procedure, the global stiffness matrix is obtained as

$$
\begin{array}{c}
\begin{array}{ccccc} 1\;2\ldots n & n+1 & n+2\;\;n+3\ldots 2n+1 & 2n+2 & 2n+3\;\;2n+4\ldots 3n+2 \end{array}\\
\left[\begin{array}{c|c|c|c|c}
\begin{matrix}[A_1]\\(n\times n)\end{matrix} & & \begin{matrix}[A_2]\\(n\times n)\end{matrix} & & \begin{matrix}[0]\\(n\times n)\end{matrix}\\
\hline
 & & & & \\
\hline
\begin{matrix}[A_2]\\(n\times n)\end{matrix} & & \begin{matrix}[A_3]\\(n\times n)\end{matrix} & & \begin{matrix}[A_4]\\(n\times n)\end{matrix}\\
\hline
 & & & & \\
\hline
\begin{matrix}[0]\\(n\times n)\end{matrix} & & \begin{matrix}[A_4]\\(n\times n)\end{matrix} & & \begin{matrix}[A_5]\\(n\times n)\end{matrix}
\end{array}\right]
\end{array}
\begin{Bmatrix}
\eta_1\\ \eta_2\\ \vdots\\ \eta_n\\ \hline \eta_{n+1}\\ \hline \eta_{n+2}\\ \eta_{n+3}\\ \vdots\\ \eta_{2n+1}\\ \hline \eta_{2n+2}\\ \hline \eta_{2n+3}\\ \eta_{2n+4}\\ \vdots\\ \eta_{3n+2}
\end{Bmatrix}
=
\begin{Bmatrix}
-F_1\\ -F_2\\ \vdots\\ -F_n\\ \hline -F_{n+}\\ \hline 0\\ 0\\ \vdots\\ 0\\ \hline F_{2n-}\\ \hline F_{2n-}\\ F_{2n-}\\ \vdots\\ F_{3n-}
\end{Bmatrix}
\tag{2.10a}
$$

$$
\text{or} \qquad [K_g]\,\{\eta\}_g = \{F\}_g \tag{2.10b}
$$

where $[K_g]$ is global stiffness matrix.

Equation (2.10) relates the forces and displacements in the finite element mesh of the repeated section. The boundary conditions of nodes (n+1) and (2n+2) are given by

$$\eta_{n+1} = \eta_{2n+2} = 0 \tag{2.11}$$

The global stiffness matrix is partitioned as indicated by dotted lines in equation (2.10a). It may be noted here that all $[A]$ matrices are symmetric, the two corner matrices are null matrices, and $[A_1]$, $[A_2]$, $[A_4]$ and $[A_5]$ are diagonal matrices. Equations (2.10a) are rewritten as

$$[A_1]\ \{\eta\}_1 + [A_2]\ \{\eta\}_2 = \{-F\}_1 \tag{2.12}$$

$$[A_2]\ \{\eta\}_1 + [A_3]\ \{\eta\}_2 + [A_4]\ \{\eta\}_3 = \{0\} \tag{2.13}$$

$$[A_4]\ \{\eta\}_2 + [A_5]\ \{\eta\}_3 = \{F\}_3 \tag{2.14}$$

where $\{\eta\}_1$, $\{\eta\}_2$ and $\{\eta\}_3$ are displacement vectors at left hand, right hand and at the joints of finite element mesh of the repeated section respectively; $\{-F\}_1$ and $\{F\}_3$ are force vectors at left hand and right hand of the section under consideration respectively.

To derive the TM for the repeated section, one needs to determine $\{\eta\}_3$ and $\{F\}_3$ in terms of $\{\eta\}_1$ and $\{F\}_1$. This can be accomplished by eliminating $\{\eta\}_2$ from the equations (2.12), (2.13) and (2.14) and is done as follows:

From equation (2.13)

$$\{\eta\}_2 = -[A_3]^{-1} [A_2] \{\eta\}_1 - [A_3]^{-1} [A_4] \{\eta\}_3 \qquad (2.15)$$

Substituting this in equation (2.12) one gets

$$\{\eta\}_3 = [T_1] \{\eta\}_1 + [T_2] \{F\}_1 \qquad (2.16)$$

where $[T_1] = [T_2]([A_1] - [A_2][A_3]^{-1} [A_2])$

and $$[T_2] = [[A_2][A_3]^{-1} [A_4]]^{-1} \qquad (2.17)$$

Substituting $\{\eta\}_2$ and $\{\eta\}_3$ from equations (2.15) and (2.16) in equation (2.14), one gets

$$\{F\}_3 = [T_3] \{\eta\}_1 + [T_4] \{F\}_1 \qquad (2.18)$$

where $[T_3] = ([A_5] - [A_4][A_3]^{-1} [A_4]) [T_1]$

$\qquad - [A_4][A_3]^{-1} [A_2]$

and $$[T_4] = ([A_5] - [A_4][A_3]^{-1} [A_4]) [T_2] \qquad (2.19)$$

Equations (2.16) and (2.18) are combined to obtain

$$\begin{Bmatrix} \{\eta\}_3 \\ \{F\}_3 \end{Bmatrix} = \begin{bmatrix} [T_1] & [T_2] \\ [T_3] & [T_4] \end{bmatrix} \begin{Bmatrix} \{\eta\}_1 \\ \{F\}_1 \end{Bmatrix} \qquad (2.20a)$$

or $$\{Z\}_3 = [T] \{Z\}_1 \qquad (2.20b)$$

where [T] is TM for the repeated section.

2.2.1 TRANSFER MATRIX FOR ORTHOGONAL CABLE NETWORK

Take the case of an m x n rectangular orthogonal cable network shown in Figure 3, where m and n are number of cables in

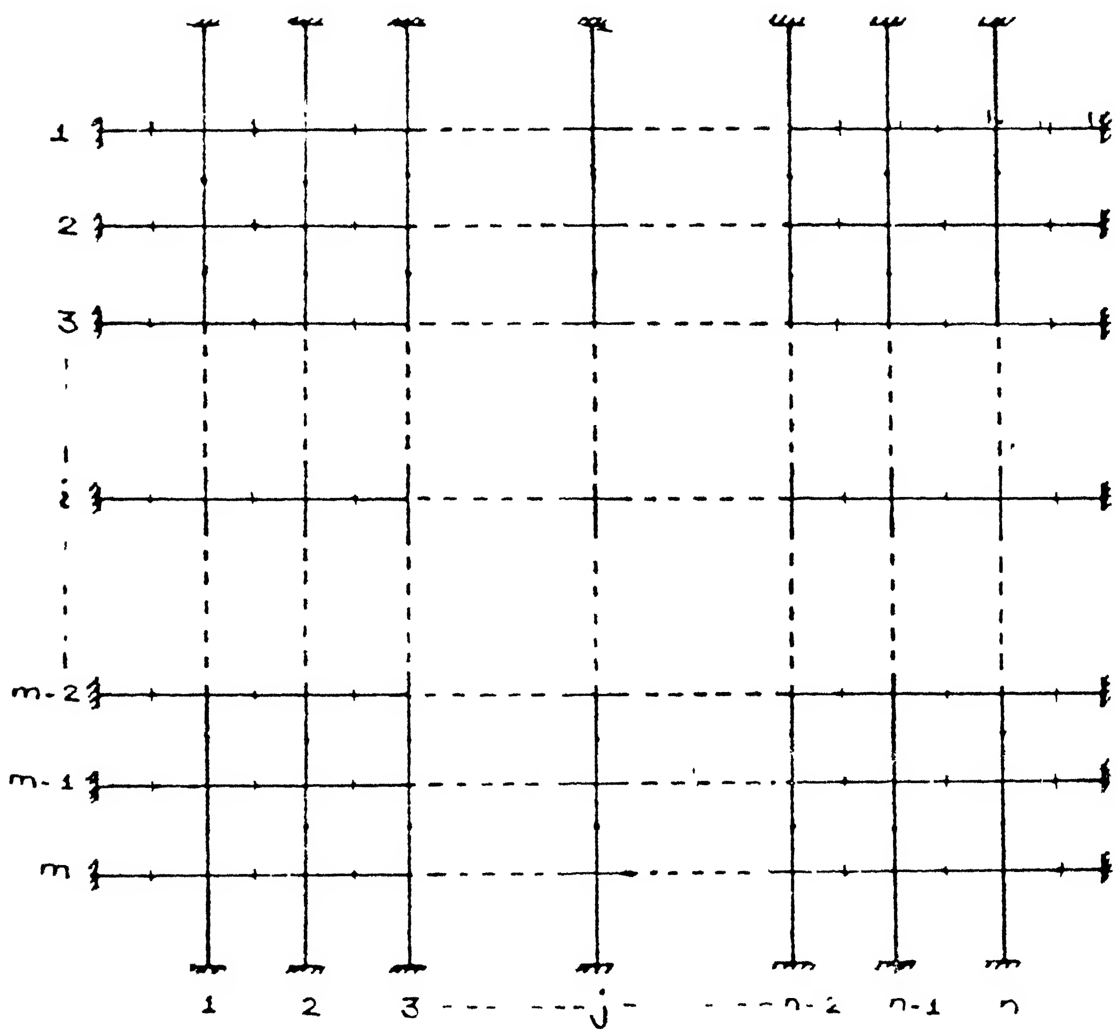


Fig. 3 m x n RECTANGULAR ORTHOGONAL CABLE NETWORK

X and Y directions respectively. The cables lying in X direction are divided into 2(n+1) equal fields, that is, portion of each cable between two adjacent joints comprises of two fields. In the case of cables lying in the Y direction, the portion of cable between two adjacent joints and at the boundaries is treated as single field. It is to be noted that equations (2.8a), (2.8b) and (2.9a), (2.9b) hold good for X and Y direction fields. For jth column repeated section shown in Figure 4a, of the orthogonal cable network, the equations (2.20a) and (2.20b) developed in Section 2.1.3 can be rewritten as

$$\begin{Bmatrix} \{\eta\}_{j+1} \\ \{F\}_{j+1} \end{Bmatrix} = \begin{bmatrix} [T_1] & [T_2] \\ [T_3] & [T_4] \end{bmatrix} \begin{Bmatrix} \{\eta\}_j \\ \{F\}_j \end{Bmatrix} \tag{2.21a}$$

or $$\{Z\}_{j+1} = [T]\,\{Z\}_j \tag{2.21b}$$

where [T] is of order 2m x 2m, $\{\eta\}_{j+1}$, $\{\eta\}_j$ and $\{F\}_{j+1}$, $\{F\}_j$ are displacement and force vectors at right hand and left hand sides of the jth column respectively.

2.2.1.1 TRANSFER MATRIX FOR LEFT END BOUNDARY OF CABLE NETWORK

For element of cable network shown in Figure 4b, TM equation (2.4a) can be rewritten as

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix}_{S_{i,1}} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix}_{S_{i,L}} \tag{2.22}$$

i.e. $$\left.\begin{aligned} \eta_{S_{i,1}} &= A_1\eta_{S_{i,L}} + A_2F_{S_{i,L}} \\ F_{S_{i,1}} &= A_3\eta_{S_{i,1}} + A_4F_{S_{i,L}} \end{aligned}\right\} \text{for } i = 1,2,\ldots,m \tag{2.23}$$

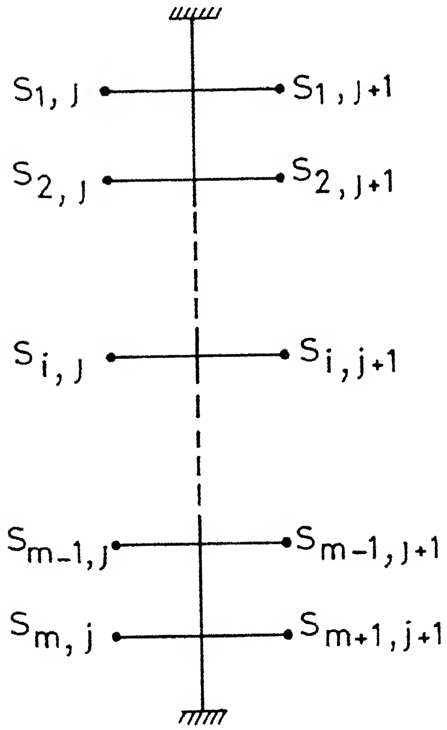


Fig. 4(a) $j^{\underline{th}}$ column of orthogonal cable

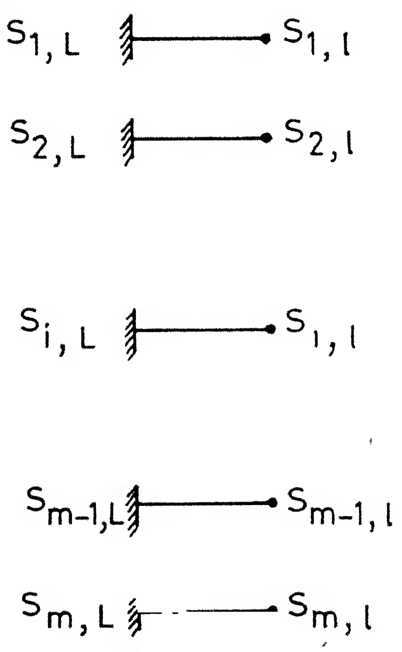


Fig. 4(b) Left end boundary of the orthogonal network.

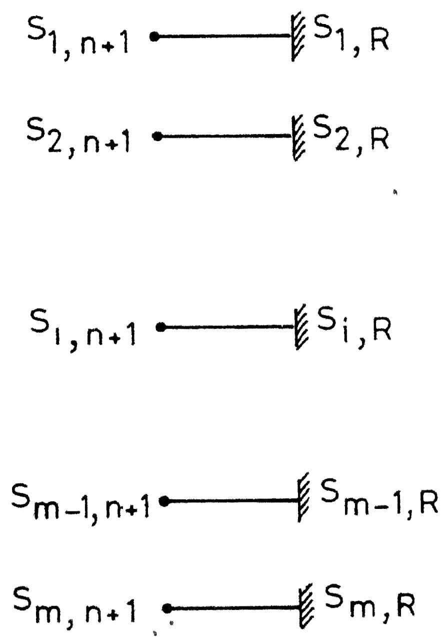


Fig. 4(c) Right end boundary of the orthogonal network.

or,

$$\{Z\}_1 = [T_L]\{Z\}_L \tag{2.24}$$

where $\{Z\}_1$ and $\{Z\}_L$ are state vectors consisting of displacements and forces for all stations $S_{i,1}$ and $S_{i,L}$, $i = 1,2,\ldots,m$ and

$$[T_L] = \begin{bmatrix} A_1 & 0 & \cdots & 0 & 0 & A_2 & 0 & \cdots & 0 & 0 \\ 0 & A_1 & \cdots & 0 & 0 & 0 & A_2 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & A_1 & 0 & 0 & 0 & \cdots & A_2 & 0 \\ 0 & 0 & \cdots & 0 & A_1 & 0 & 0 & \cdots & 0 & A_2 \\ A_3 & 0 & \cdots & 0 & 0 & A_4 & 0 & \cdots & 0 & 0 \\ 0 & A_3 & \cdots & 0 & 0 & 0 & A_4 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & A_3 & 0 & 0 & 0 & \cdots & A_4 & 0 \\ 0 & 0 & \cdots & 0 & A_3 & 0 & 0 & \cdots & 0 & A_4 \end{bmatrix} \tag{2.25}$$

2.2.1.2 TRANSFER MATRIX FOR RIGHT END BOUNDARY OF CABLE NETWORK

For element of cable network shown in Figure 4c, TM equation (2.4a) can be rewritten as

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix}_{S_{i,R}} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix}_{S_{i,n+1}} \tag{2.26}$$

i.e.

$$\left.\begin{aligned} \eta_{S_{i,R}} &= B_1\eta_{S_{i,n+1}} + B_2F_{S_{i,n+1}} \\ F_{S_{i,R}} &= B_3\eta_{S_{i,n+1}} + B_4F_{S_{i,n+1}} \end{aligned}\right\} \text{for } i = 1,2,\ldots,m \tag{2.27}$$

or,

$$\{Z\}_R = [T_R]\{Z\}_{n+1} \tag{2.28}$$

where $\{Z\}_R$ and $\{Z\}_{n+1}$ are state vectors consisting of displacements and forces for all stations $S_{i,R}$ and $S_{i,n+1}$, $i = 1,2,\ldots,m$ and

$$[T_R] = \begin{bmatrix} B_1 & 0 & \cdots & 0 & 0 & B_2 & 0 & \cdots & 0 & 0 \\ 0 & B_1 & \cdots & 0 & 0 & 0 & B_2 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & B_1 & 0 & 0 & 0 & \cdots & B_2 & 0 \\ 0 & 0 & \cdots & 0 & B_1 & 0 & 0 & \cdots & 0 & B_2 \\ B_3 & 0 & \cdots & 0 & 0 & B_4 & 0 & \cdots & 0 & 0 \\ 0 & B_3 & \cdots & 0 & 0 & 0 & B_4 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & B_3 & 0 & 0 & 0 & \cdots & B_4 & 0 \\ 0 & 0 & \cdots & 0 & B_3 & 0 & 0 & \cdots & 0 & B_4 \end{bmatrix} \tag{2.29}$$

2.2.1.3 TRANSFER MATRIX FOR THE SYSTEM

As mentioned earlier, the system is modelled to be combination of left end boundary, n repeated sections and right end boundary section. For each of these, TM has already been obtained. Combining these in accordance with the geometry of the system, one gets

$$\{Z\}_R = [U]\{Z\}_L$$

where $[U] = [T_R][T]\ldots[T][T_L]$

$$= [T_R][T]^n[T_L] \tag{2.30}$$

Equation (2.30) is the combined TM relation between the state vectors on the right end and left end boundary of the cable network.

2.2.2 TRANSFER MATRIX FOR NON-ORTHOGONAL CABLE NETWORK

In this section the equations (2.21a) and (2.21b), are used to develop the TM for the non-orthogonal cable network. Two types of non-orthogonal networks are considered:

(i) A network in which the cables lying in the Y direction ~~starts~~ start and end on X edges is referred to as type A non-orthogonal network, Figure 5.

(ii) A network in which the cables lying in the Y direction start from X edge and end on Y edge and vice-versa, is referred to as type B non-orthogonal network, Figure 6.

2.2.2.1 TYPE A NON-ORTHOGONAL CABLE NETWORK

Figure 5 shows a square non-orthogonal type A cable network. The angle between the cables in the two directions is $\tan^{-1} L/P$, where L is the length ~~area~~ of the square network and P is the pitch between the cables. In type A square network if there are m cables in the X direction, then there will be m+1 cables in the Y direction. For the cables lying in X direction, the portion of the cables between two successive joints and at boundaries are divided into two fields of equal length, for the cables lying in Y direction, the portion of the cables between two successive joints and at boundaries are treated as a single field.

Equations (2.26a) and (2.26b), derived for the orthogonal cable networks are applicable to the type A network. For

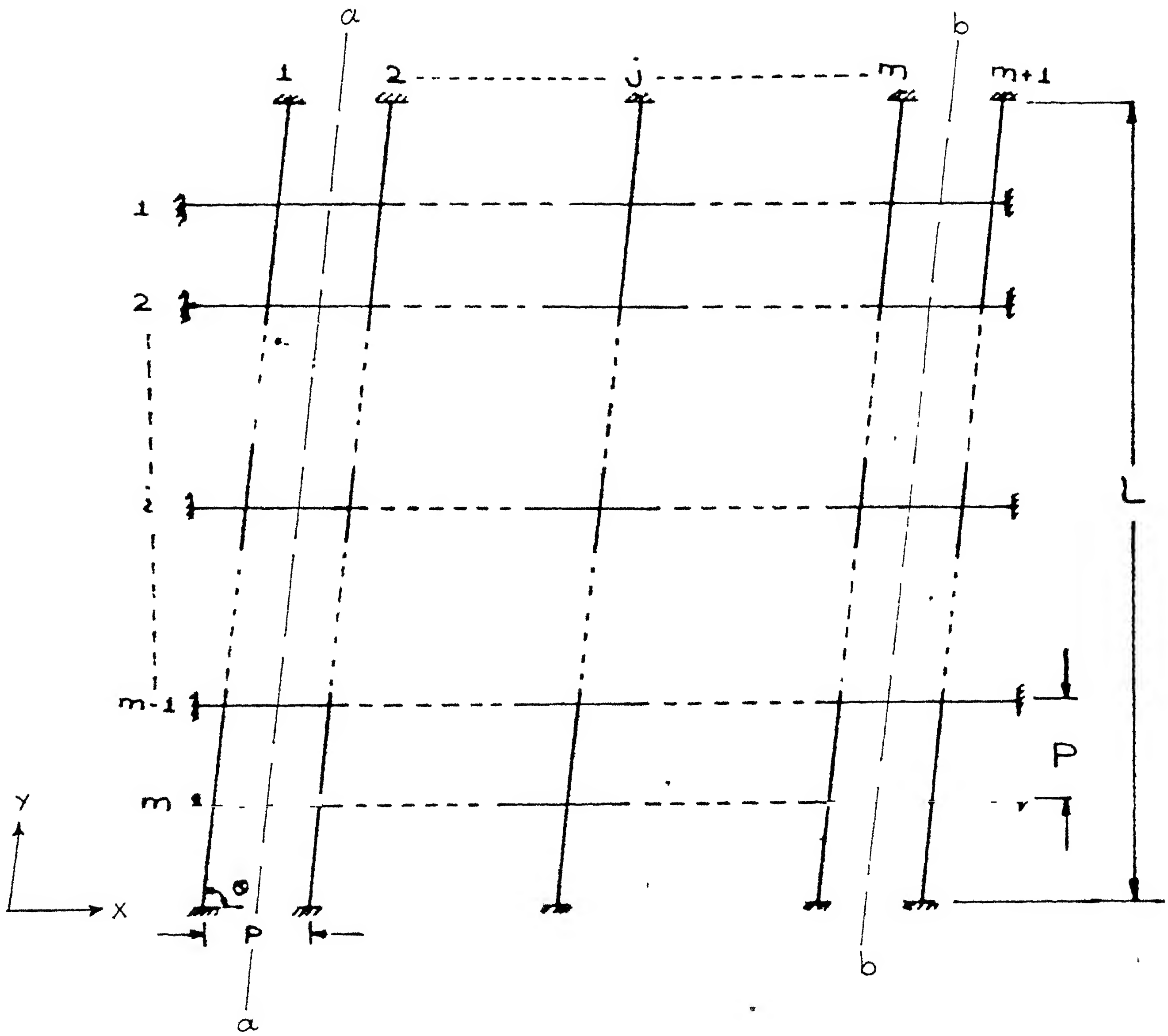


Fig. 5 TYPE A SQUARE NON-ORTHOGONAL CABLE NETWORK.
$\theta = \tan^{-1} L/P$

a set of cables to the left of a-a, Figure 5, let $[T_1]$ be the TM. The matrix $[T_1]$ is similar to the matrix $[T]$ of equation (2.21b) with the difference that the stiffness matrix $[K_1]$ used for the field in the X direction (left side of the first cable in the Y direction $(j = 1)$) is

$$[K_1] = \begin{bmatrix} p\,\frac{\cos pX}{\sin pX} & -\,\frac{p}{\underset{\text{sin}}{\cancel{\text{six}}}\,pX} \\ -\,\frac{p}{\sin pX} & p\,\frac{\cos pX}{\sin pX} \end{bmatrix} \qquad (2.31)$$

where $\quad X = (m + 1 - i)/(m + 1) \qquad (2.32)$

and i is for the ith cable in the X direction. Similarly $[T_2]$ is the TM for cables to the right of section b-b, Figure 5. The stiffness matrix $[K_1]$ used for the field in the X direction (right side of the cable in the Y direction $(j = m + 1)$)is

$$[K_1] = \begin{bmatrix} p\,\frac{\cos pX}{\sin pX} & -\,\frac{p}{\sin pX} \\ -\,\frac{p}{\sin pX} & p\,\frac{\cos pX}{\sin pX} \end{bmatrix} \qquad (2.33)$$

where $\quad X = i/(m + 1) \qquad (2.34)$

Keeping in view the geometry of the network the combined TM is obtained as

$$\begin{aligned} \{Z\}_R &= [T_2]\{Z\}_{m+1} = [T_2][T] \ldots [T][T_1]\{Z\}_L \\ &= [T_2][T]^{m-1}[T_1]\{Z\}_L \\ &= [U]\{Z\}_L \end{aligned}$$

where $\quad [U] = [T_2][T]^{m-1}[T_1] \qquad (2.35)$

2.2.2.2 TYPE B NON-ORTHOGONAL CABLE NETWORK

In this chapter transfer matrix for the orthogonal and non-orthogonal type A cable networks has been developed relating the state vectors at left end and right end boundaries. In case of type B non-orthogonal cable networks, the system is modelled in such a way that the combined TM relate the state vector of all the stations where Y-directed cable terminate and top and right hand boundaries to the state vector of all the stations from where Y-directed cable starts at bottom and left end boundaries, Figure 6. This is done in the following manner:

Firstly state vector $\{Z\}_{S_1}$ at section S_1 as shown in Figure 7c, is obtained in terms of state vector $\{Z\}_B$ at bottom boundary. The state vector $\{Z\}_{S_2}$ at section S_2 can be obtained from state vector $\{Z\}_{S_1}$ by a TM which is generated by adding elements of TM for the Y-directed cable element between stations $S_{1,L}$ and $S_{2,m-1}$ to that of a repeated section TM $[T_o]$ marked by thick lines. To proceed further, the state vector $\{Z\}_{S_3}$ at section S_3 can be derived by substracting the terms for Y-directed cable element lying between the stations $S_{2,2m-1}$ and $S_{2,R}$ from state vector $\{Z\}_{S_2}$ and then multiplying by TM $[T_o]$ and finally adding terms for Y-directed cable element as done earlier. Proceeding in the similar fashion TM relating state vectors for stations along top and right end boundaries, and left and bottom end boundaries can be obtained. This requires TM^s for fields below and above any horizontal cable i which is obtainable from equation (2.6a) as

$$\begin{Bmatrix} \eta \\ F \end{Bmatrix} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \begin{Bmatrix} \eta \\ F \end{Bmatrix} \qquad (2.36)$$

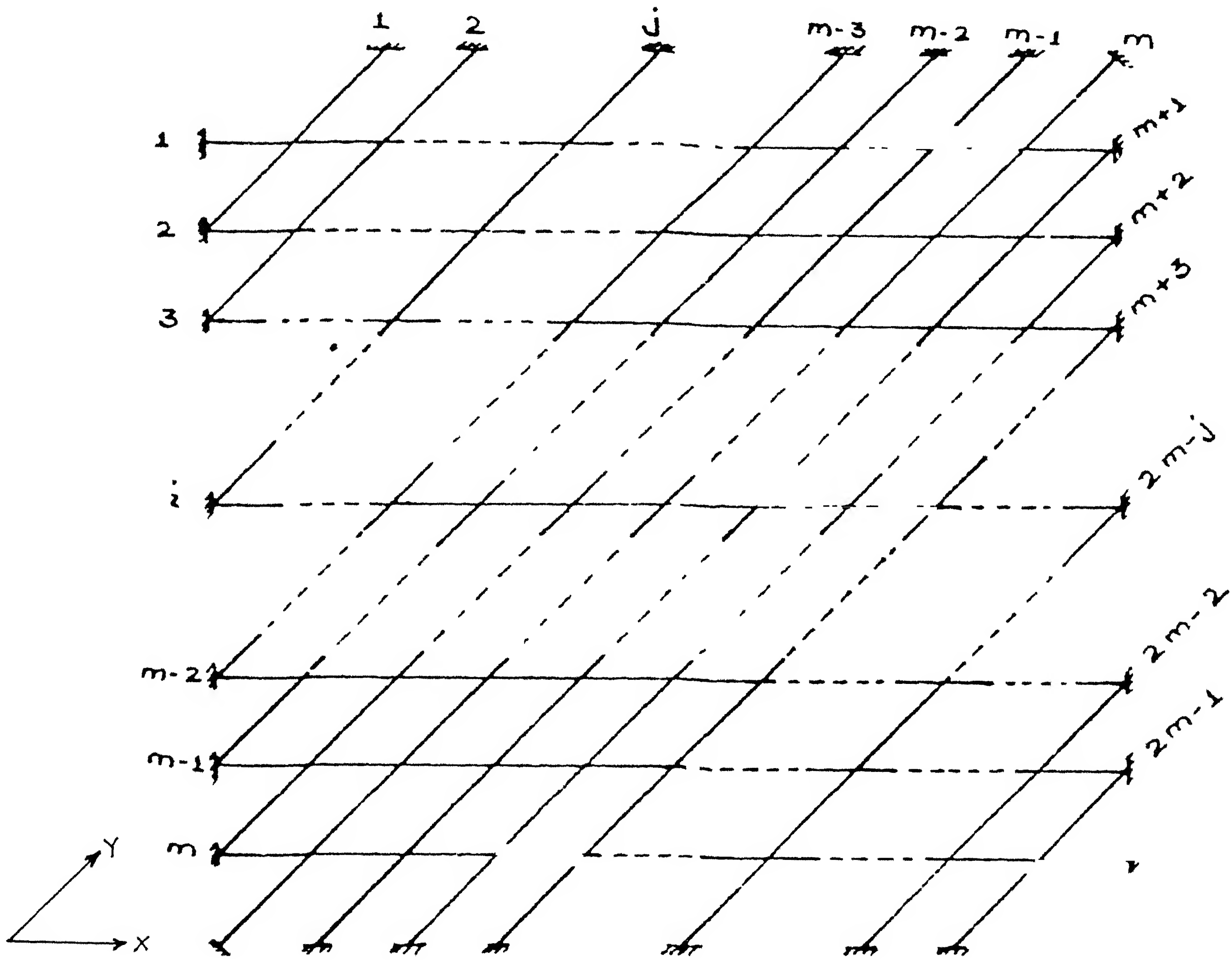


Fig. 6 TYPE B SQUARE NON-ORTHOGONAL CABLE NETWORK.
$\theta = \pi/4$

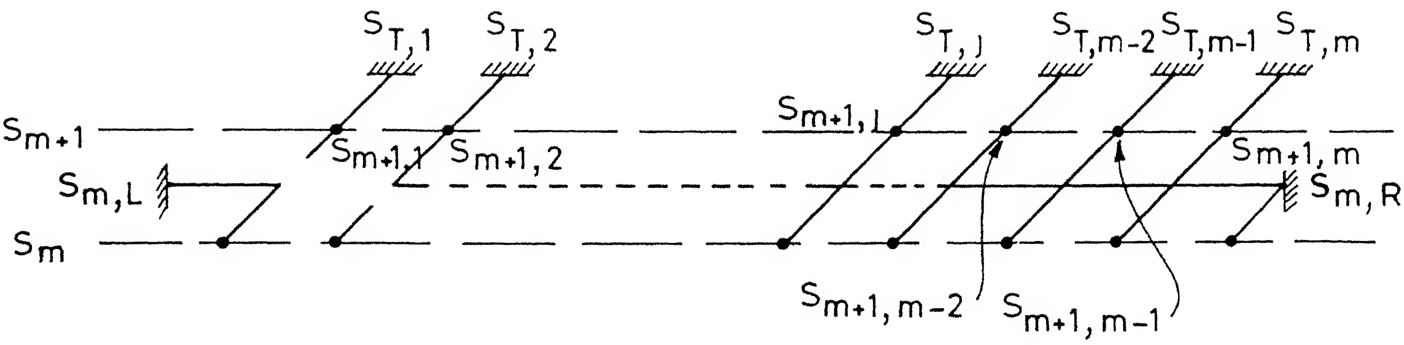


Fig. 7(a) Top boundary and last row of type-B nonorthogonal cable network.

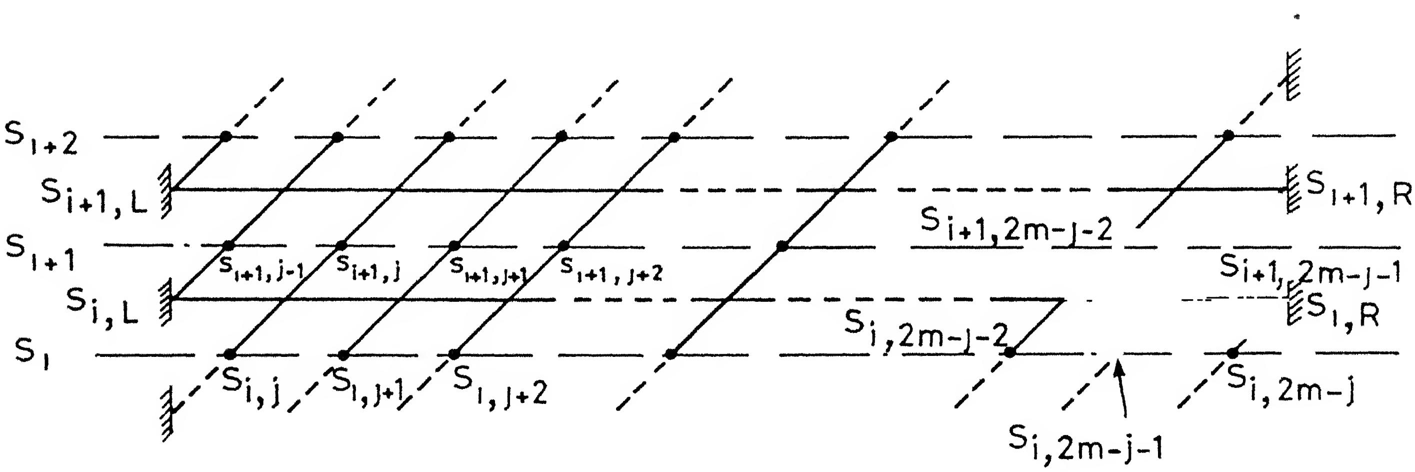


Fig. 7(b) $i^{\underline{th}}$ and $(i+1)^{\underline{th}}$ row of type-B nonorthogonal cable network.

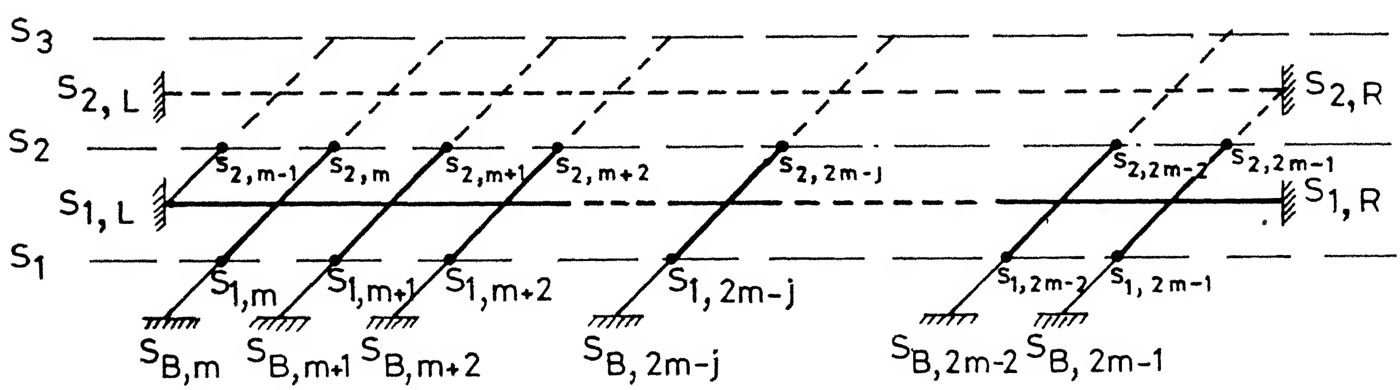


Fig. 7(c) Bottom boundary and first row of type B non-orthogonal cable network.

and

$$\begin{Bmatrix} \eta \\ \\ F \end{Bmatrix}_{S_{i+1,j}} = \begin{bmatrix} D_1 & D_2 \\ \\ D_3 & D_4 \end{bmatrix} \begin{Bmatrix} \eta \\ \\ F \end{Bmatrix}_{S_J} \tag{2.37}$$

respectively. It may be noted that suffix J stands for a joint.

The state vector $\{Z\}_{S_1}$ can be expressed in terms of state vector $\{Z\}_B$ in the following way:

$$\{Z\}_{S_1} = [T_B]\,\{Z\}_B \tag{2.38}$$

where $[T_B]$ is TM for bottom end boundary. The TM $[T_B]$ is the same as expression equation (2.25) except that its elements are the elements of TM of equation (2.36). Proceeding as stated above

$$\{Z\}_{S_2} = [TI_{22}]\,\{Z\}_{B1} \tag{2.39}$$

where $\{Z\}_{S_2}$ is state vector at S_2

$\{Z\}_{B1}$ is state vector comprising of displacements and forces at stations $S_{1,L}$ and bottom end boundary

$$[TI_{22}] = \begin{bmatrix} D_1 & 0 \;.\;.\;.\; 0 & D_2 & 0 \;.\;.\;.\; 0 \\ \begin{matrix}0\\ \vdots \\ 0\end{matrix} & \begin{bmatrix} \\ TI_1(11) \\ \\ \end{bmatrix} & \begin{matrix}0\\ \vdots \\ 0\end{matrix} & \begin{bmatrix} \\ TI_1(12) \\ \\ \end{bmatrix} \\ D_3 & 0 \;.\;.\;.\; 0 & D_4 & 0 \;.\;.\;.\; 0 \\ \begin{matrix}0\\ \vdots \\ 0\end{matrix} & \begin{bmatrix} \\ TI_1(21) \\ \\ \end{bmatrix} & \begin{matrix}0\\ \vdots \\ 0\end{matrix} & \begin{bmatrix} \\ TI_1(22) \\ \\ \end{bmatrix} \end{bmatrix} \tag{2.40}$$

$$[TI_1] = [T_o][T_B] \tag{2.41}$$

and

$$[TI_1] = \left[\begin{array}{c:c} TI_1(11) & TI_1(12) \\ \hdashline TI_1(21) & TI_1(22) \end{array}\right] \tag{2.42}$$

To arrive at state vector $\{Z\}_{S_3}$ at section S_3 two rows namely (m+1) and (2m+1) are eliminated from equation (2.39) in order to drop the Y-directed cable element between stations $S_{2,2m-1}$ and $S_{2,R}$, thus

$$\{Z\}'_{S_2} = [TI_{32}] \{Z\}_{B1} \tag{2.43}$$

It may be noted that $[TI_{32}]$ can be obtained from $[TI_{22}]$ of equation (2.40) by dropping the (m+1)th row and last row. The order of the TM $[TI_{32}]$ is (2m) x (2m+2). Proceeding on the same lines, one arrives at

$$\{Z\}_{S_{m+1}} = [TI_{2(m+1)}] \{Z\}_{LB} \tag{2.44}$$

where $\{Z\}_{S_{m+1}}$ is state vector at section S_{m+1}

$\{Z\}_{LB}$ is state vector at stations comprising of bottom and left boundary except for station $S_{m,L}$.

The order of TM $[TI_{2(m+1)}]$ is (2m) x (4m-2).

To arrive at $\{Z\}_T$ for top boundary of the cable network, $\{Z\}_{S_{m+1}}$ is multiplied by TM $[T_T]$ for the top boundary, thus

$$\{Z\}_T = [T_T] \{Z\}_{S_{m+1}} \tag{2.45}$$

$$= [T_T] [TI_{2(m+1)}] \{Z\}_{LB} \tag{2.46}$$

or $$\{Z\}_T = [U'] \{Z\}_{LB} \tag{2.47}$$

Denoting $\{Z\}_{TR}$ as state vector for the set of stations at top and right hand boundary except $S_{1,R}$ and $\{Z\}_{LB}$ for the

set of stations at left and bottom of the network except $S_{m+1,L}$

$$\{Z\}_{TR} = \lfloor U \rfloor \{Z\}_{LB} \tag{2.48}$$

where $[U\rfloor$ is generated by adding elements of TM of the right hand boundary fields (except for station $S_{1,R}$) to $[U']$. These are the elements which were dropped earlier from matrices $[TI_{2i}]$, $i = 2,3,\ldots,m$.

2.3 CHARACTERISTIC EQUATION

The boundary conditions at left and right ends of the cable network are:

$$\{\eta\}_L = \{0\} \quad \text{and} \quad \{\eta\}_R = \{0\} \tag{2.49}$$

respectively. Substitution of these boundary conditions in the combined TM relation provides the characteristic equation. The combined TM relation for orthogonal and non-orthogonal type A cable network is

$$\{Z\}_R = [U]\{Z\}_L$$

On partitioning, the displacements and forces, one gets

$$\left\{\begin{array}{c}\eta \\ \hline F\end{array}\right\}_R = \left[\begin{array}{c|c} U_1 & U_2 \\ \hline U_2 & U_4 \end{array}\right] \left\{\begin{array}{c}\eta \\ \hline F\end{array}\right\}_L \tag{2.50}$$

where $[U_1]$, $[U_2]$, $[U_3]$ and $[U_4]$ are square matrices of order m. Applying the boundary conditions of equation (2.49), one gets

$$\left\{\begin{array}{c}0 \\ \hline F\end{array}\right\}_R = \left[\begin{array}{c|c} U_1 & U_2 \\ \hline U_3 & U_4 \end{array}\right] \left\{\begin{array}{c}0 \\ \hline F\end{array}\right\}_L \tag{2.51}$$

or $\quad [U_2]\{F\}_L = \{0\}$ (2.52)

The natural frequencies of the cable networks are determined from zeros of the above characteristic equation. Similarly, the combined TM relation for non-orthogonal type B cable network is

$$\{Z\}_{TR} = [U]\{Z\}_{LB}$$

On partitioning, one has

$$\begin{Bmatrix} \eta \\ \hline F \end{Bmatrix}_{TR} = \left[\begin{array}{c|c} U_1 & U_2 \\ \hline U_3 & U_4 \end{array}\right] \begin{Bmatrix} \eta \\ \hline F \end{Bmatrix}_{LB} \qquad (2.53)$$

Here the boundary conditions at left and right ends, and top and bottom ends are

$$\{\eta\}_L = \{0\}, \quad \{\eta\}_R = \{0\}, \quad \{\eta\}_T = \{0\} \quad \text{and} \quad \{\eta\}_B = \{0\} \qquad (2.54)$$

respectively. Applying boundary conditions of equations (2.54), one gets

$$\begin{Bmatrix} 0 \\ \hline F \end{Bmatrix}_{TR} = \left[\begin{array}{c|c} U_1 & U_2 \\ \hline U_3 & U_4 \end{array}\right] \begin{Bmatrix} 0 \\ \hline F \end{Bmatrix}_{LB} \qquad (2.55)$$

or $\quad [U_2]\{F\}_{LB} = \{0\}$ (2.56)

For its non-trivial solution, one must have

$$|U_2| = 0$$

the zeros of which lead to the natural frequencies.

CHAPTER 3

VIBRATION ANALYSIS OF BEAM GRILLAGE

In the last chapter transfer matrices have been developed for the transverse vibration of various types of cable networks. In this chapter FETM technique will be developed for the analysis of a beam grillage. As discussed in the case of cable network, orthogonal beam grillage also has the repeated characteristic. Figure 10a shows the repeated section of a orthogonal beam grillage. If the TM relating the state vector on left hand side and right hand side is known for a repeated section, the combined TM can be obtained, according to the geometry of the grillage.

The grillage is analysed for its transverse vibration for (a) combined bending-torsional mode and (b) bending mode only by suitably idealizing the joints between the crossing member or adopting suitable stiffness properties. Rotary inertia and shear effects in the transverse vibration of the grillage are neglected, i.e. a beam element is idealized as an Euler's beam.

Firstly the TM^s for the transverse and torsional vibration of an individual (X- or Y-directed) member are written and then these are combined to from the TM of the repeated section of beam grillage. By multiplying the transfer matrices of repeated section and that of left end and right end boundaries, the combined TM [U] is generated. Using the boundary conditions the characteristic equation is arrived at which is solved for its roots.

3.1 TRANSFER MATRICES FOR INDIVIDUAL BEAM ELEMENT

The governing equations of motion for transverse and torsional vibration of a beam element, shown in Figure 8a are [23]

$$EI \frac{\partial^4 W}{\partial X^4} = - m \frac{\partial^2 W}{\partial t^2} \tag{3.1}$$

and

$$C \frac{\partial^2 \theta}{\partial X^2} = \rho I_p \frac{\partial^2 \theta}{\partial t^2} \tag{3.2}$$

respectively, where

- E is modulus of elasticity
- I is area moment of inertia
- m is mass per unit length
- ρ is density
- $C = GI_{pc}$ = torsional rigidity
- G is shear modulus
- I_{pc} is polar moment of inertia of equivalent circular section
- I_p is polar moment of inertia of rectangular section.

Its TM for the transverse vibration is given by [19]

$$\begin{Bmatrix} -W \\ \Psi \\ M \\ V \end{Bmatrix}_R = \begin{bmatrix} C_o & LC_1 & \frac{L^2}{EI} C_2 & \frac{L^3}{EI} C_3 \\ p^4 \frac{C_3}{L} & C_o & \frac{L}{EI} C_1 & \frac{L^2}{EI} C_2 \\ p^4 \frac{EI}{L^2} C_2 & p^4 \frac{EI}{L} C_3 & C_o & LC_1 \\ p^4 \frac{EI}{L^3} C_1 & p^4 \frac{EI}{L^2} C_2 & p^4 \frac{C_3}{L} & C_o \end{bmatrix} \begin{Bmatrix} -W \\ \Psi \\ M \\ V \end{Bmatrix}_L \tag{3.3}$$

where $C_0 = (\cosh p + \cos p)/2$

$C_1 = (\sinh p + \sin p)/(2p)$

$C_2 = (\cosh p - \cos p)/(2p^2)$

$C_3 = (\sinh p - \sin p)/(2p^3)$

$$p^4 = m\omega^2 L^4/(EI) \tag{3.4}$$

and ω is angular frequency of vibration.

The TM for the torsional vibrations of the beam element, shown in Figure 8a, is given by [19]

$$\begin{Bmatrix} \theta \\ T \end{Bmatrix}_R = \begin{bmatrix} \cos\beta & \frac{L}{GI_{pc}}\frac{\sin\beta}{\beta} \\ -\frac{GI_{pc}}{L}\frac{\sin\beta}{\beta} & \cos\beta \end{bmatrix} \begin{Bmatrix} \theta \\ T \end{Bmatrix}_L \tag{3.5}$$

where

$$\beta = \omega L((I_p\rho)/(GI_{pc}))^{1/2}$$

The grillage has beam elements in X and Y direction and as such their respective variables, material and geometric properties are identified by the suffixes X and Y. The equations (3.1) to (3.5) are non-dimensionalised with respect to the variables and material properties of the member lying in Y direction and are denoted by bar ($\bar{\ }$) and are given below. Thus TM for Y direction and X direction members take the shape as follows.

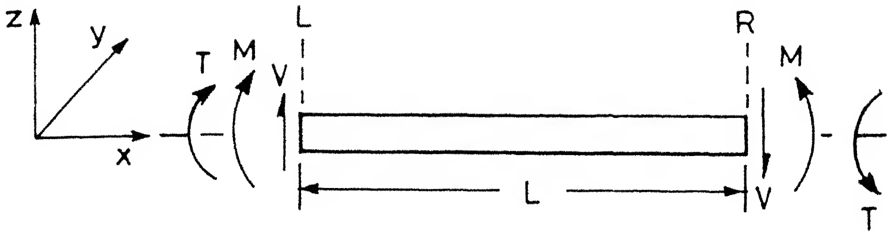


Fig. 8(a) A beam element

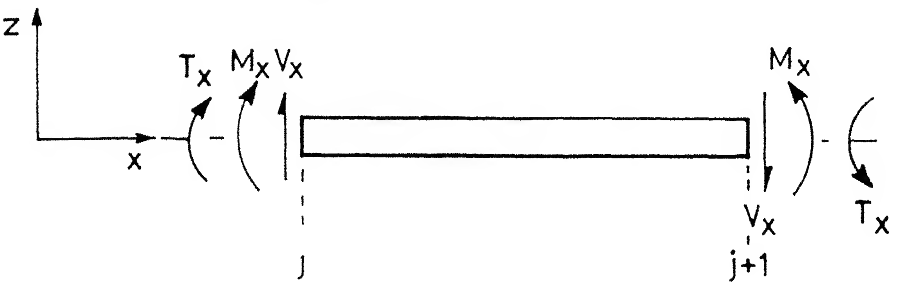


Fig. 8(b) X-directed beam element

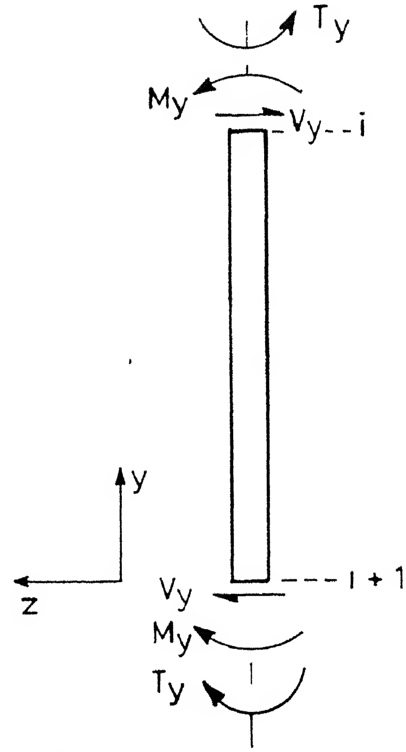


Fig. 8(c) Y-directed beam element.

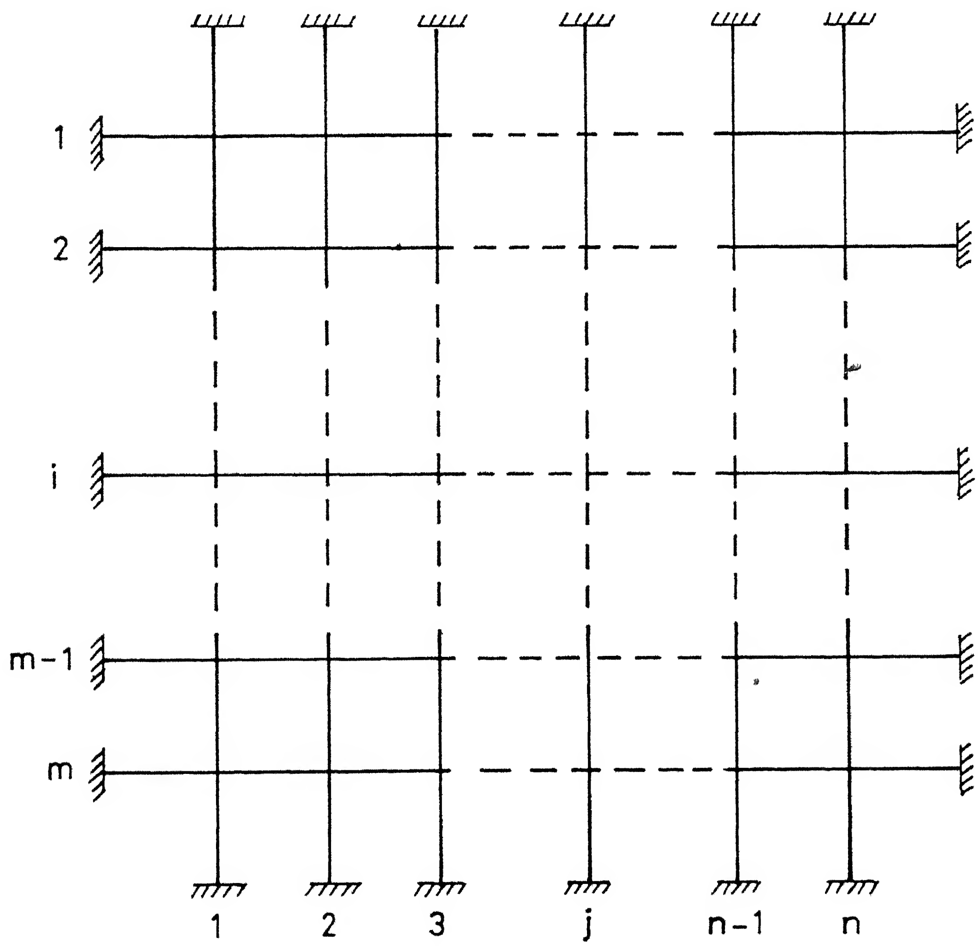


Fig. 9 A m x n orthogonal beam grillage.

Transfer Matrix for Transverse Vibration of Y-directed Beam Element:

$$\begin{Bmatrix} -\overline{W}_y \\ \overline{\Psi}_y \\ \overline{M}_y \\ \overline{V}_y \end{Bmatrix}_{i+1} = \begin{bmatrix} C_o & C_1 & C_2 & C_3 \\ p^4C_3 & C_o & C_1 & C_2 \\ p^4C_2 & p^4C_3 & C_o & C_1 \\ p^4C_1 & p^4C_2 & p^4C_3 & C_o \end{bmatrix} \begin{Bmatrix} -\overline{W}_y \\ \overline{\Psi}_y \\ \overline{M}_y \\ \overline{V}_y \end{Bmatrix}_i \tag{3.6}$$

where $\overline{W}_y = W_y/L_y$, $\overline{\Psi}_y = \Psi_y$, $\overline{M}_y = (M_yL_y)/(E_yI_y)$

$$\overline{V}_y = (V_yL_y^2)/(E_yI_y), \quad p^4 = (m_yL_y^4\omega^2)/(E_yI_y) \tag{3.7}$$

Transfer Matrix for Torsional Vibrations of Y-Directed Beam Element:

$$\begin{Bmatrix} \overline{\theta}_y \\ \overline{T}_y \end{Bmatrix}_{i+1} = \begin{bmatrix} \cos p_1 & \frac{1}{r_1p_1}\sin p_1 \\ -r_1p_1\sin p_1 & \cos p_1 \end{bmatrix} \begin{Bmatrix} \overline{\theta}_y \\ \overline{T}_y \end{Bmatrix}_i \tag{3.8}$$

where $\overline{\theta}_y = \theta_y$, $\overline{T}_y = (T_yL_y)/(E_yI_y)$,

$$p_1 = \omega((\rho_yI_{py}L_y^2)/(G_yI_{pcy}))^{1/2}, \quad r_1 = (I_{pcy}G_y)/(E_yI_y) \tag{3.9}$$

Transfer Matrix for Transverse Vibrations of X-directed Beam Element:

$$\begin{Bmatrix} -\overline{W}_x \\ \overline{\Psi}_x \\ \overline{M}_x \\ \overline{V}_x \end{Bmatrix}_{j+1} = \begin{bmatrix} D_o & \eta D_1 & \frac{\eta^2}{r_2} D_2 & \frac{\eta^3}{r_2} D_3 \\ \frac{p_2^4}{\eta} D_3 & D_o & \frac{\eta}{r_2} D_1 & \frac{\eta^2}{r_2} D_2 \\ \frac{p_2^4 r_2}{\eta^2} D_2 & \frac{p_2^4 r_2}{\eta} D_3 & D_o & \eta D_1 \\ \frac{p_2^4 r_2}{\eta^3} D_1 & \frac{p_2^4 r_2}{\eta^2} D_2 & \frac{p_2^4}{\eta} D_3 & D_o \end{bmatrix} \begin{Bmatrix} -\overline{W}_x \\ \overline{\Psi}_x \\ \overline{M}_x \\ \overline{V}_x \end{Bmatrix}_j \tag{3.10}$$

$$\begin{aligned} \text{where} \quad D_o &= (\cosh p_2 + \cos p_2)/2 \\ D_1 &= (\sinh p_2 + \sin p_2)/(2p_2) \\ D_2 &= (\cosh p_2 - \cos p_2)/(2p_2^2) \\ D_3 &= (\sinh p_2 - \sin p_2)/(2p_2^3) \end{aligned} \tag{3.11}$$

$$\overline{W}_x = W_x/L_y, \quad \eta = L_x/L_y, \quad \overline{M}_x = (M_x L_y)/(E_y I_y)$$

$$\overline{V}_x = (V_x L_y^2)/(E_y I_y), \quad r_2 = (E_x I_x)/(E_y I_y)$$

$$p_2^4 = (m_x L_x^4 \omega^2)/(E_x I_x) \tag{3.12}$$

Transfer Matrix for Torsional Vibrations of X-directed Beam Element:

$$\begin{Bmatrix} \overline{\Theta}_x \\ \overline{T}_x \end{Bmatrix}_{j+1} = \begin{bmatrix} \cos p_3 & \frac{\eta}{r_3 p_3} \sin p_3 \\ -\frac{r_3 p_3}{\eta} \times \sin p_3 & \cos p_3 \end{bmatrix} \begin{Bmatrix} \overline{\Theta}_x \\ \overline{T}_x \end{Bmatrix}_j \tag{3.13}$$

where $\bar{\theta}_x = \theta_x$, $T_x = (T_x L_y)/(E_y I_y)$

$$p_3 = \omega((\rho_x I_{px} L_y^2)/(G_x I_{pcx}))^{1/2}(L_x/L_y),$$

$$r_3 = (I_{pcx} G_x)/(E_y I_y) \qquad (3.14)$$

3.2 TRANSFER MATRIX FOR A REPEATED SECTION OF BEAM GRILLAGE IN COMBINED BENDING-TORSIONAL VIBRATIONS

The transfer matrix for a repeated section of a cable network has been developed in Section 2.1.3. On the similar lines, the TM for beam grillage can be developed. ~~to~~ To do so, one requires to determine the stiffness matrix for a beam element.

Using equations (3.6) and (3.8), state vector $\{Z\}_{i+1}$ at section i+1, Figure 8c, can be written as in terms of state vector $\{Z\}_i$ at section i as

$$\{Z\}_{i+1} = [T_1]\ \{Z\}_i \qquad (3.15)$$

$$\{Z\}_{i+1} = \begin{Bmatrix} \bar{\theta}_y \\ -\bar{W}_y \\ \bar{\Psi}_y \\ \bar{M}_y \\ \bar{V}_y \\ \bar{T}_y \end{Bmatrix}_{i+1} \quad \text{and} \quad \{Z\}_i = \begin{Bmatrix} \bar{\theta}_y \\ -\bar{W}_y \\ \bar{\Psi}_y \\ \bar{M}_y \\ \bar{V}_y \\ \bar{T}_y \end{Bmatrix}_i$$

To express forces in terms of displacements, equation (3.15) can be rewritten as

$$\begin{Bmatrix} \{D_y\} \\ \hline \{F_y\} \end{Bmatrix}_{i+1} = \left[\begin{array}{c|c} T_{111} & T_{112} \\ \hline T_{121} & T_{122} \end{array}\right] \begin{Bmatrix} \{D_y\} \\ \hline \{F_y\} \end{Bmatrix}_i \qquad (3.16)$$

where

$$\{D\} = \begin{Bmatrix} \bar{\Theta} \\ -\bar{W} \\ \bar{\Psi} \end{Bmatrix} \quad \text{and} \quad \{F\} = \begin{Bmatrix} \bar{M} \\ \bar{V} \\ \bar{T} \end{Bmatrix}$$

Expanding equation (3.16) in terms of displacement and force vectors, force vectors $\{F_y\}_{i+1}$ and $\{F_y\}_i$ can be expressed in terms of displacement vectors $\{D_y\}_{i+1}$ and $\{D_y\}_i$ as

$$\begin{Bmatrix} -\{F_y\}_i \\ \{F_y\}_{i+1} \end{Bmatrix} = \begin{bmatrix} [T_{112}]^{-1}[T_{111}] & -[T_{112}]^{-1} \\ [T_{121}]-[T_{122}][T_{112}]^{-1}[T_{111}] & [T_{122}][T_{112}]^{-1} \end{bmatrix} \begin{Bmatrix} \{D_y\}_i \\ \{D_y\}_{i+1} \end{Bmatrix}$$

(3.17a)

$$\text{or} \qquad \{F\}_y = [K_1^{(e)}]\,\{D\}_y \qquad (3.17b)$$

Similarly, for a beam element lying in the X direction, equation (3.17) can be rewritten as

$$\begin{Bmatrix} -\{F_x\}_j \\ \{F_x\}_{j+1} \end{Bmatrix} = \begin{bmatrix} [T_{212}]^{-1}[T_{211}] & -[T_{212}]^{-1} \\ [T_{221}]-[T_{222}][T_{212}]^{-1}[T_{211}] & [T_{222}][T_{212}]^{-1} \end{bmatrix} \begin{Bmatrix} \{D_x\}_j \\ \{D_x\}_{j+1} \end{Bmatrix}$$

(3.18a)

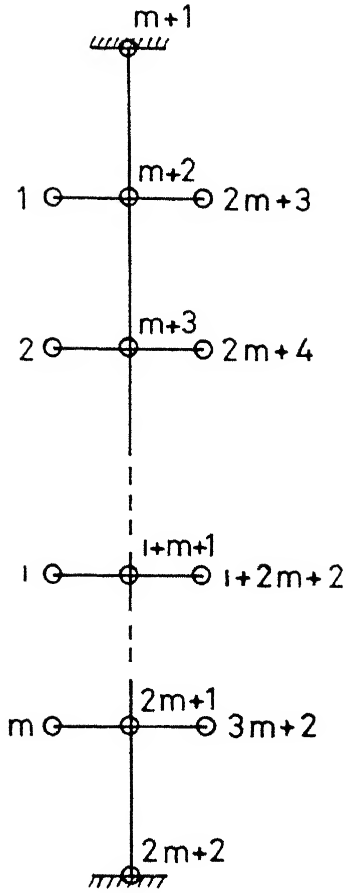
$$\text{or} \qquad \{F\}_x = [K_2^{(e)}]\,\{D\}_x \qquad (3.18b)$$

$[K_1^{(e)}]$ and $[K_2^{(e)}]$ are stiffness matrices of order (6x6) for X- and Y-directed beam elements respectively.

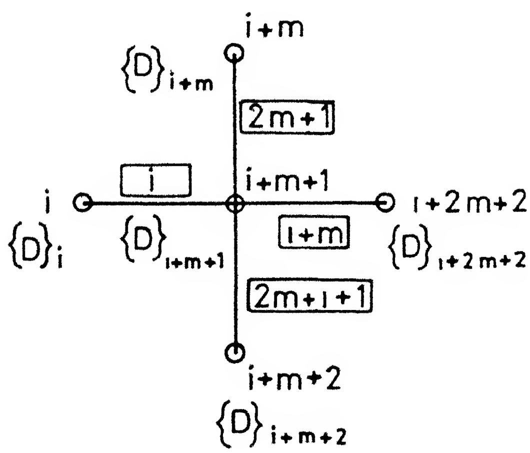
Figure 10a shows a repeated section of the beam grillage as for cable network and the numbering system adopted for fields in the section. The joint (i+m+1) is shown separately to explain the assemblage of i, i+m along the X direction and (2m+i), (2m+i+1) along the Y direction. For these fields the state vectors consisting of displacement vectors $\begin{Bmatrix} \{D^{(i)}\}_1 \\ \{D^{(i)}\}_2 \end{Bmatrix}$ have been labelled with superscript of the number of element and subscripts of its two ends and are

$$\begin{Bmatrix} \begin{Bmatrix} \bar{\Theta}_x^{(i)} \\ -\bar{W}_x^{(i)} \\ \bar{\Psi}_x^{(i)} \end{Bmatrix}_1 \\ \begin{Bmatrix} \bar{\Theta}_x^{(i)} \\ -\bar{W}_x^{(i)} \\ \bar{\Psi}_x^{(i)} \end{Bmatrix}_2 \end{Bmatrix}, \quad \begin{Bmatrix} \begin{Bmatrix} \bar{\Theta}_x^{(i+m)} \\ -\bar{W}_x^{(i+m)} \\ \bar{\Psi}_x^{(i+m)} \end{Bmatrix}_1 \\ \begin{Bmatrix} \bar{\Theta}_x^{(i+m)} \\ -\bar{W}_x^{(i+m)} \\ \bar{\Psi}_x^{(i+m)} \end{Bmatrix}_2 \end{Bmatrix}, \quad \begin{Bmatrix} \begin{Bmatrix} \bar{\Theta}_y^{(2m+i)} \\ -\bar{W}_y^{(2m+i)} \\ \bar{\Psi}_y^{(2m+i)} \end{Bmatrix}_1 \\ \begin{Bmatrix} \bar{\Theta}_y^{(2m+i)} \\ -\bar{W}_y^{(2m+i)} \\ \bar{\Psi}_y^{(2m+i)} \end{Bmatrix}_2 \end{Bmatrix}, \quad \begin{Bmatrix} \begin{Bmatrix} \bar{\Theta}_y^{(2m+i+1)} \\ -\bar{W}_y^{(2m+i+1)} \\ \bar{\Psi}_y^{(2m+i+1)} \end{Bmatrix}_1 \\ \begin{Bmatrix} \bar{\Theta}_y^{(2m+i+1)} \\ -\bar{W}_y^{(2m+i+1)} \\ \bar{\Psi}_y^{(2m+i+1)} \end{Bmatrix}_2 \end{Bmatrix}$$

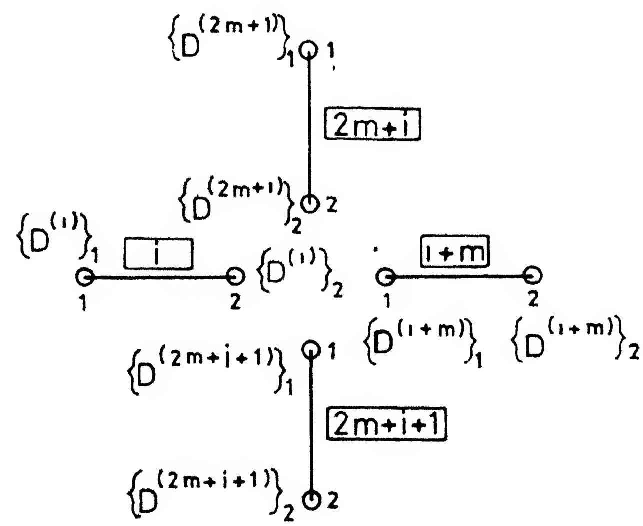
Since for the repeated section having a number of fields and joints, the state vector consisting of displacements has to have a global numbering systems. In view of this above mentioned state vectors for elements meeting at (i+m+1), are designated respectively as



(a) A repeated section



(b) $(i+m+1)^{\underline{th}}$ joint



(c) Elements meeting at $(i+m+1)^{\underline{th}}$ joint

Fig. 10 Finite element idealisation of a repeated section of beam grillage.

$$\left\{\begin{array}{c}\begin{Bmatrix}\bar{\Psi}_y\\-\bar{W}\\\bar{\Psi}_x\end{Bmatrix}_i\\\\\begin{Bmatrix}\bar{\Psi}_y\\-\bar{W}\\\bar{\Psi}_x\end{Bmatrix}_{i+m+1}\end{array}\right\},\quad\left\{\begin{array}{c}\begin{Bmatrix}\bar{\Psi}_y\\-\bar{W}\\\bar{\Psi}_x\end{Bmatrix}_{i+m+1}\\\\\begin{Bmatrix}\bar{\Psi}_y\\-\bar{W}\\\bar{\Psi}_x\end{Bmatrix}_{i+2m+2}\end{array}\right\},\quad\left\{\begin{array}{c}\begin{Bmatrix}\bar{\Psi}_x\\-\bar{W}\\\bar{\Psi}_y\end{Bmatrix}_{i+m}\\\\\begin{Bmatrix}\bar{\Psi}_x\\-\bar{W}\\\bar{\Psi}_y\end{Bmatrix}_{i+m+1}\end{array}\right\},\quad\left\{\begin{array}{c}\begin{Bmatrix}\bar{\Psi}_x\\-\bar{W}\\\bar{\Psi}_y\end{Bmatrix}_{i+m+1}\\\\\begin{Bmatrix}\bar{\Psi}_x\\-\bar{W}\\\bar{\Psi}_y\end{Bmatrix}_{i+m+2}\end{array}\right\}$$

Having obtained the stiffness matrices for X- and Y-directed beam elements, these are assembled on lines similar to that of FEM as shown in equation (3.19).

Equation (3 19)

1 2 m 1 2 m 1 2 .m | m+1 m+1 m+1 | m+2 m+3 2m+1 m+2 m+3 2m+1 m+2 m+3 2m+1 | 2m+2 2m+2 2m+2 | 2m+3 2m+4 3m+2 2m+3 [illegible] [illegible] [illegible] [illegible] 3m+2

$$
\left[\begin{array}{c|c|c|c|c}
[A_1]_{3m\times 3m} & & [A_2]_{3m\times 3m} & & [\text{illegible}]_{3m\times 3m} \\ \hline
 & & & & \\ \hline
[A_0]_{3m\times 3m} & & [A_3]_{3m\times 3m} & & [A_4]_{3m\times 3m} \\ \hline
 & & & & \\ \hline
[0]_{3m\times 3m} & & [A_7]_{3m\times 3m} & & [A_5]_{3m\times 3m}
\end{array}\right]
\left\{\begin{array}{c}
\bar\Psi_{y1} \\ \bar\Psi_{y2} \\ \bar\Psi_{ym} \\ -\bar\eta_1 \\ -\bar\eta_2 \\ -\bar W_m \\ \bar\Psi_{x1} \\ \bar\Psi_{x2} \\ \bar\Psi_{xm} \\ \hline
\bar\Psi_{ym+1} \\ -\bar W_{m+1} \\ \bar\Psi_{xm+1} \\ \hline
\bar\Psi_{ym+2} \\ \bar\Psi_{ym+3} \\ \bar\Psi_{y2m+1} \\ -\bar W_{m+2} \\ -\bar\eta_{m+3} \\ -\bar\eta_{2m+1} \\ \bar\Psi_{xm+2} \\ \bar\Psi_{xm+3} \\ \bar\Psi_{x2m+1} \\ \hline
\bar\Psi_{y2m+2} \\ -\bar\eta_{2m+2} \\ \bar\Psi_{x2m+2} \\ \hline
\bar\Psi_{y2m+3} \\ \bar\Psi_{y2m+4} \\ \bar\Psi_{y3m+2} \\ -\bar W_{2m+3} \\ -\bar\eta_{2m+4} \\ -\bar\eta_{3m+2} \\ \bar\Psi_{x2m+3} \\ \bar\Psi_{x2m+4} \\ \bar\Psi_{x3m+2}
\end{array}\right\}
=
\left\{\begin{array}{c}
-\bar M_{y1} \\ -\bar M_{y2} \\ -\bar M_{ym} \\ -\bar V_1 \\ -\bar V_2 \\ -\bar V_m \\ -\bar M_{x1} \\ -\bar M_{x2} \\ -\bar M_{xm} \\ \hline
\bar M_{ym+1} \\ \bar V_{m+1} \\ \bar M_{xm+1} \\ \hline
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline
\bar M_{y2m+2} \\ \bar V_{2m+2} \\ \bar M_{x2m+2} \\ \hline
\bar M_{y2m+3} \\ \bar M_{y2m+4} \\ \bar M_{y3m+2} \\ \bar V_{2m+3} \\ \bar V_{2m+4} \\ \bar V_{3m+2} \\ \bar M_{x2m+3} \\ \bar M_{x2m+4} \\ \bar M_{x3m+2}
\end{array}\right\}
$$

Equation (3.19) relates the forces and displacement vectors in the finite element mesh of the repeated section of the beam grillage column as shown in Figure 10a. To transform equation (3.19) in the standard form of TM, boundary conditions of nodes (m+1) and (2m+2)

are applied. For the clamped case these are

$$\bar{\Psi}_{yk} = \bar{W}_k = \bar{\Psi}_{xk} = 0 \tag{3.20}$$

where k =(m+1) and (2m+2).

Then equation (3.19) is reduced to the standard form of TMM as done for the cable network i.e.

$$\begin{Bmatrix} \{D\}_3 \\ \{F\}_3 \end{Bmatrix} = \begin{bmatrix} [T_1] & [T_2] \\ [T_3] & [T_4] \end{bmatrix} \begin{Bmatrix} \{D\}_1 \\ \{F\}_1 \end{Bmatrix} \tag{3.21a}$$

or
$$\{Z\}_3 = [T]\{Z\}_1 \tag{3.21b}$$

where {D} and {F} are displacement and force vectors respectively and

$$[T_1] = [T_2]([A_1] - [A_2][A_3]^{-1}[A_6])$$

$$[T_2] = \lfloor [A_2][A_3]^{-1}[A_4] \rfloor^{-1}$$

$$[T_3] = ([A_5] - [A_7][A_3]^{-1}[A_4])[T_1] - [A_7][A_3]^{-1}[A_6]$$

$$[T_4] = ([A_5] - [A_7][A_3]^{-1}[A_4])[T_2] \tag{3.22}$$

Suffix 1 is used for left hand side of grillage finite element mesh and 3 is used for right hand side.

3.3 TRANSFER MATRIX FOR BEAM GRILLAGE WITH CLAMPED ENDS

The TM for the system can be generated corresoonding to the geometry of the grid on lines similar to the cable network proceeding from the left hand end, where

$$\{Z\}_1 = [T_L]\{Z\}_L \qquad (3.23)$$

the state vector for (j+1) repeated section is given by

$$\{Z\}_{j+1} = [T]^j [T_L] \{Z\}_L \qquad (3.24)$$

where $j = 1,\ldots,n$.

and finally for the right hand end as

$$\{Z\}_R = [T_R][T]^n [T_L] \{Z\}_L$$

$$\text{or} \quad \{Z\}_R = [U] \{Z\}_L \qquad (3.25)$$

where $[U] = [T_R][T]^n [T_L]$

Relation (3.25) is the combined TM relation between the state vector on right end and left end of beam grillage. Note that the clamped conditions of the lower and upper horizontal ends have already been incorporated in the formulation.

3.4 CHARACTERISTIC EQUATION

For the beam grillage with clamped ends the characteristic equation can be derived using boundary conditions at left and right ends, similar to that of cable network. The boundary conditions for these ends are

$$\{D\}_L = \{0\} \quad \text{and} \quad \{D\}_R = \{0\} \qquad (3.26)$$

where

$$\{D\} = \begin{Bmatrix} \bar{\Psi}_y \\ -\bar{V} \\ \bar{\Psi}_x \end{Bmatrix}.$$

On partitioning equation (3.25), one has

$$\begin{Bmatrix} \{D\}_R \\ \hline \{F\}_R \end{Bmatrix} = \left[\begin{array}{c|c} U_1 & U_2 \\ \hline U_3 & U_4 \end{array}\right] \begin{Bmatrix} \{D\}_L \\ \hline \{F\}_L \end{Bmatrix} \qquad (3.27)$$

Now applying the above mentioned boundary conditions

$$\lfloor U_2 \rfloor \{F\}_L = \{0\} \qquad (3.28)$$

For its non-trivial solution one must have

$$|U_2| = 0 \qquad (3.29)$$

and the natural frequencies of the system are determined from the zeros of this equation.

3.5 MODE SHAPES

In Sections 3.2 and 3.3 the TM for the repeated section, left hand boundary and right hand boundary have been developed. These matrices will be used to develop the mode shape.

Firstly, the homogeneous equation (3.28) is solved to compute $\{F\}_L$. Since $\{D\}_L$ is a null vector, so $\{Z\}_L$ is completely known. Multiplying $\{Z\}_L$ by $[T_L]$, it gives $\{Z\}_1$ i.e. state vector on the left hand side of Ist column. From $\{Z\}_1$, the displacement vector $\{D\}_1$ can be found. Again by multiplying $[T]$ (TM for repeated section), $\{Z\}_2$ can be computed. In this way, the state vectors on the left hand and right hand side of the beam column

lying in the Y direction can be known.

To find out the displacement vector $\{D\}_J$ at joints, $\{Z\}$ at joints is determined as following:

Let $\{D\}_J$ be displacement vector at the left hand of the jth column and $\{D\}_{J+1}$ at the right hand of the jth column. Then by using equation (3.19)

$$\{D\}_J = - [A_3]^{-1} [[A_6] \{D\}_J + [A_4] \{D\}_{J+1}] \qquad (3.30)$$

Knowing the displacement vector at the different sections and joints the mode shape can be plotted.

In the case of repeated roots, the rank of equation (3.28) is reduced by two. This is solved by assigning values to two elements of the vector $\{F\}_L$. It is quite likely that though two modal vectors are orthogonal to other modal vectors corresponding to the distinct roots, but are not mutually orthogonal. These are orthogonalised by standard procedure.

Transverse vibration of beam grillage in pure bending is a special case of combined bending-torsion. As such the same procedure is adopted to find out the nodal frequencies and the mode shape by deleting terms corresponding to the torsional vibration.

CHAPTER 4

RESULTS

In Chapters 2 and 3, the TM^{s} for the orthogonal and non-orthogonal cable networks and orthogonal beam grillage have been developed. In this chapter, the method of solution is presented so as to compute its natural frequencies. Modes for the beam grillage have also been found. The natural frequencies for various cable networks are compared with those obtained by TMM [21] and FEM [7], whereas those of the orthogonal beam grillage are compared with the FEM [18] and the plate analogy of beam grillage [18].

4.1 METHOD OF SOLUTION

The natural frequencies of the system are determined from zeros of the characteristic equation $|U_2| = 0$. Newton-Raphson modified method is used to compute the zeros of characteristic equation.

Pramod Gupta [21] used Golden Section Method for computation of natural frequencies. By using this method, he minimized the function. The Golden Section method is not fast converging and a large number of trials are conducted.

For the fast convergence, Newton-Raphson modified method has been used in the present case. Since the function is not explicitly expressable, the Newton-Raphson method without modification cannot be used. To find out the tangent at a point consider two points, in its neighbourhood on its sides

and the line joining these two can be approximated as a tangent to the polynomial function. It was conjectured that for a given error of convergence, this method does not miss the double roots.

Computer programmes are developed for the computation of the natural frequencies of networks and grillage and for mode shapes for the grillage.

4.2 SPECIFICATION OF CABLE NETWORKS

The following data is used to calculate the natural frequencies of the cable network:

1.	Arm of the square cable network	:	3.048 m
2.	Cross-sectional area of the cable	:	322.58×10^{-6} m^2
3.	Weight density of the material of cable	:	0.7855126×10^5 N/m^3
4.	Tension in cables lying in X direction	:	333.673 kN
5.	Tension in cables lying in Y direction	:	333.673 kN

4.3 RESULTS FOR ORTHOGONAL CABLE NETWORKS

First four natural frequencies, important from the point of their design, of the orthogonal square networks (2x2 to 6x6), Figure 11, are calculated for the data given in Section 4.2 and are given in Table 1. These values are compared with those obtained by FEM [7] and TMM [21].

The data shows the closeness of the results. It is worth pointing out that the present results are from one exact formulation and within the limit of computational errors. Whereas in the case of FEM, the assumed displacement pattern

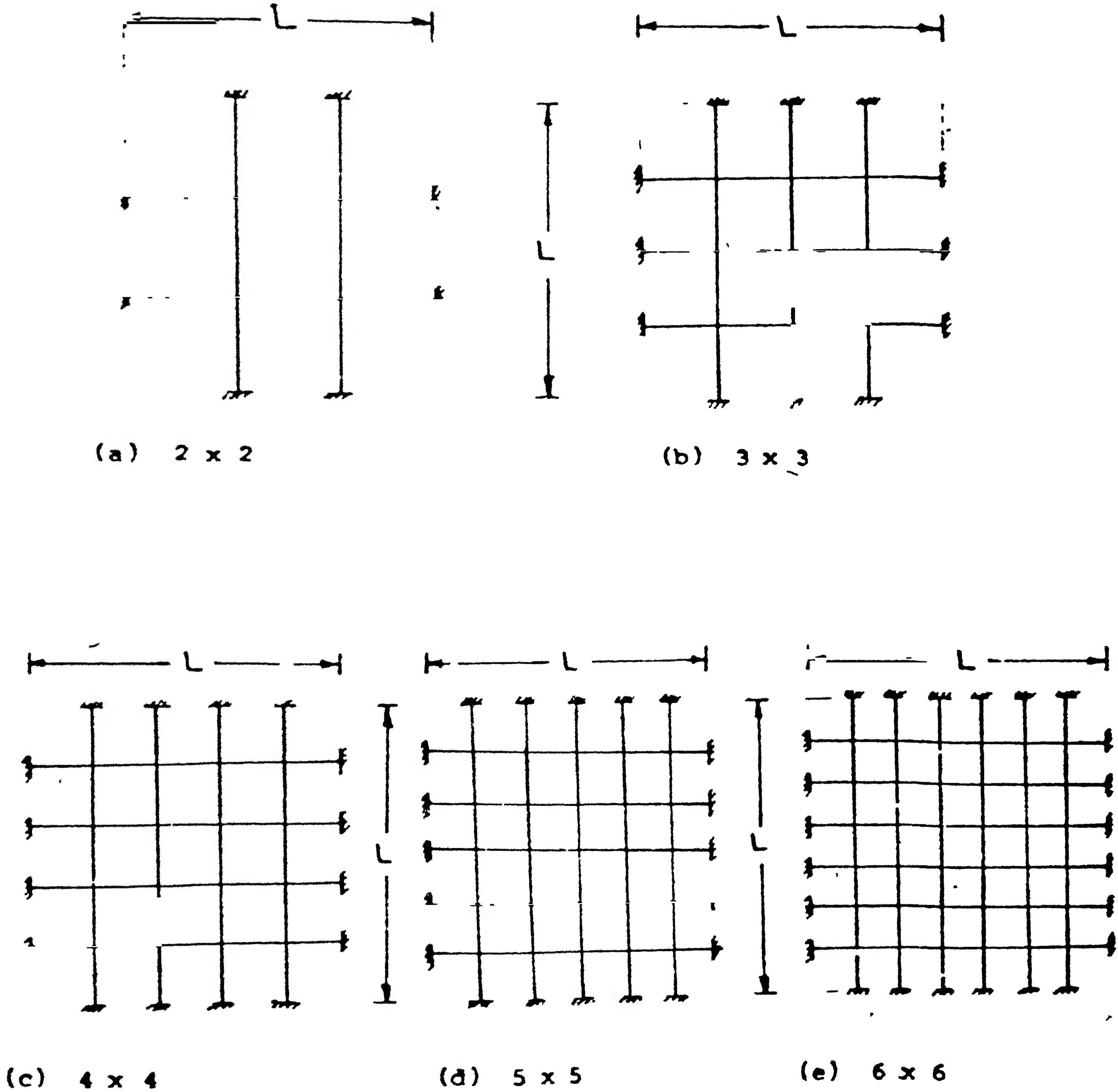


FIG 11 DIFFERENT SQUARE ORTHOGONAL CABLE NETWORKS OF THE SAME SIDE LENGTH

Table 1

[illegible] frequencies (cps) for the orthogonal square [illegible] networks

a. 2x2 Net

Mode	FETM (present method)	FEM [7]	TMM [21]
A_{11}	[illegible]	61.62	58.91
A_{12}	[illegible]	97.43	88.35
A_{21}	[illegible]	97.43	88.35
A_{22}	117.85	127.79	117.81

b. 3x3 Net

Mode	FETM (present method)	FEM [7]	TMM [21]
A_{11}	58.92	60.45	58.91
A_{12}	90.74	96.29	90.70
A_{21}	90.74	96.29	90.70
A_{22}	117.85	132.81	117.81

c. 4x4 Net

Mode	FETM (present method)	FEM [7]	TMM [21]
A_{11}	58.92	59.88	58.91
A_{12}	91.69	95.34	91.64
A_{21}	91.69	95.34	91.64
A_{22}	117.85	-	117.81

d. 5x5 Net

Mode	FETM (present method)	FEM [7]	TMM [21]
A_{11}	58.92	59.59	58.91
A_{12}	92.18	94.73	92.12
A_{21}	92.18	94.73	92.12
A_{22}	117.85	-	117.81

e. 6x6 Net

Mode	FETM (present method)	FEM [7]	TMM [21]
A_{11}	58.92	59.41	58.91
A_{12}	92.46	94.33	92.40
A_{21}	92.46	94.33	92.40
A_{22}	117.85	124.72	117.81

leads to over estimation of stiffness and hence frequencies. Since the error in assumed displacement pattern becomes more pronounced in the higher modes the error in the estimation of frequencies increases with the higher modes. Therefore, the present method is better suited and more useful for computing the frequencies and the modes. Further incorporating the FEM with the TMM the formulation has been simplified, and the results are quite comparable.

4.4 RESULTS FOR NON-ORTHOGONAL CABLE NETWORKS

In this section results for both Type A and Type B for non-orthogonal square cable networks are presented. Natural frequencies for 2x3, 3x4, 4x5 and 5x6 Type A networks, shown in Figure 12, are given in Table 2 and compared, wherever possible with those obtained by FEM [7] and TMM [21]. Figure 13 shows 2x3, 3x5, 4x7 and 5x9 Type B networks and their natural frequencies are given in Table 3. The results are comparable with those of FEM and TMM.

4.5 SPECIFICATION OF BEAM GRILLAGE

The following data is used to calculate the natural frequencies and mode shapes of the beam grillage:

1.	Length of beam in X and Y direction	:	1.27 m
2.	Area of cross section of beams in X and Y direction	:	1.93548×10^{-3} m^2
3.	Moments of inertia of the beams lying in X and Y direction	:	1.04057×10^{-7} m^4
4.	Elastic constants for the material of beams lying in X and Y direction	:	207.0×10^9 N/m^2

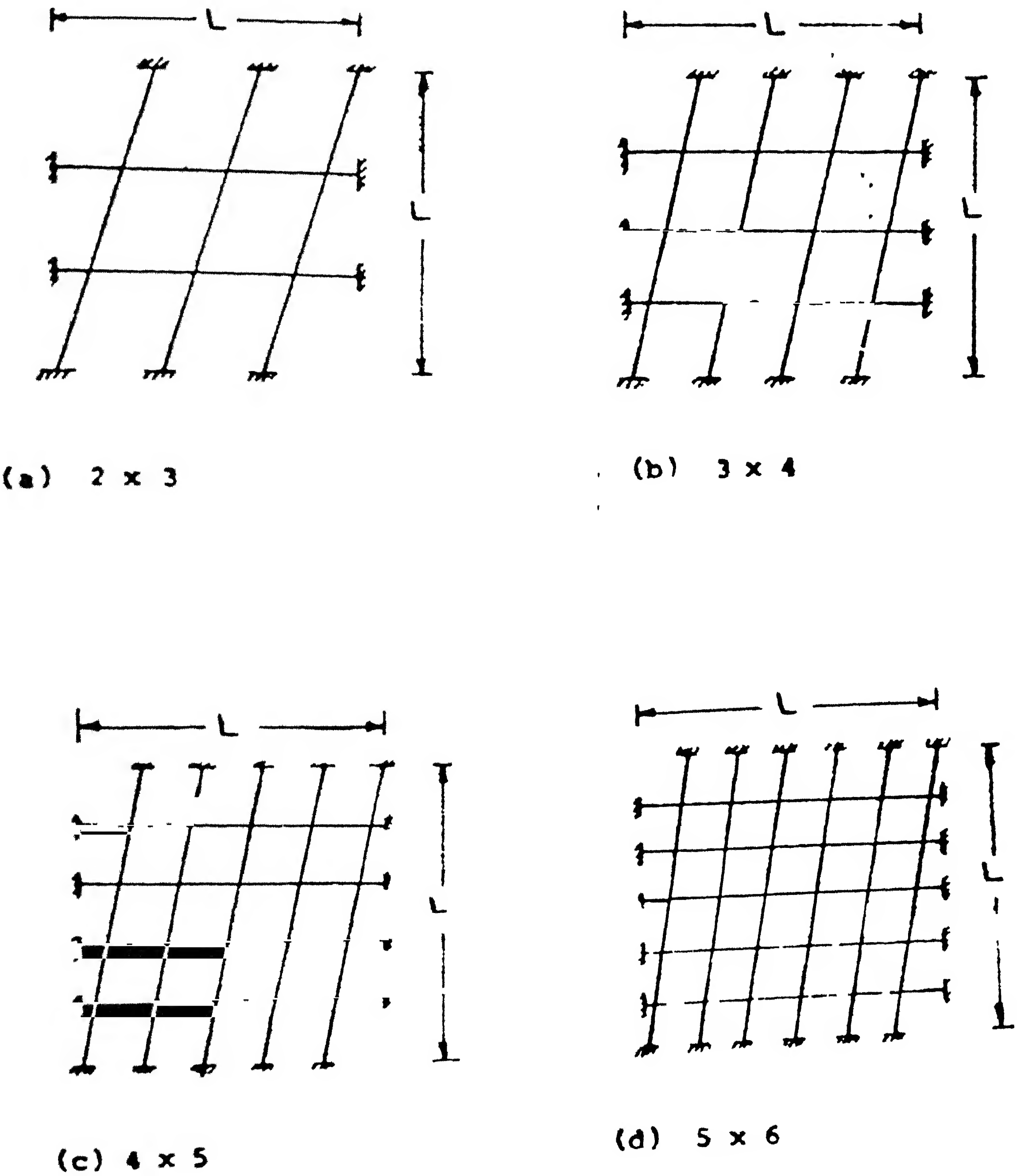
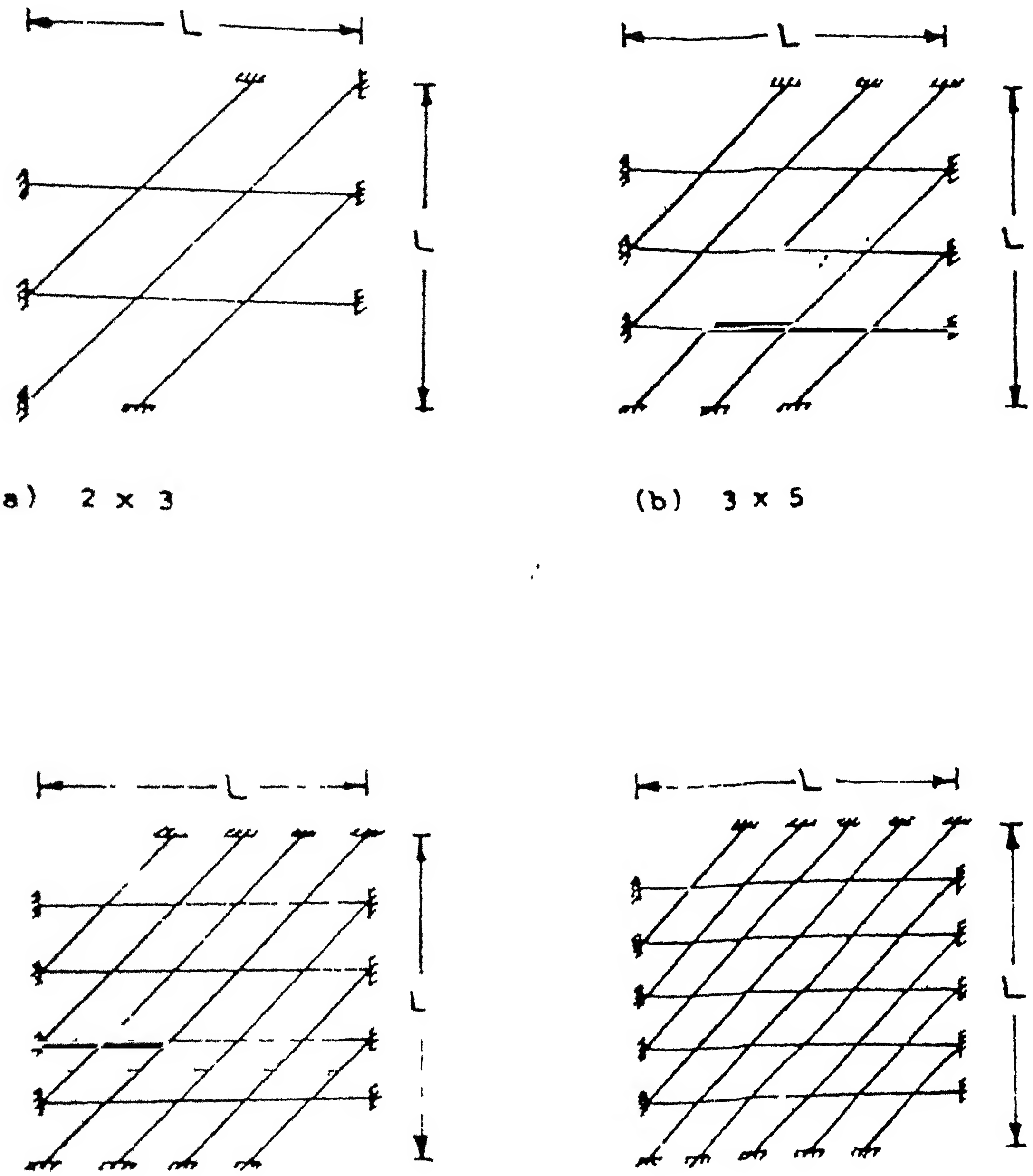


FIG 12 DIFFERENT TYPE-A NONORTHOGONAL CABLE NETWORKS OF THE SAME SIDE LENGTH



(a) 2 x 3

(b) 3 x 5

(c) 4 x 7

(d) 5 x 9

FIG 13 DIFFERENT TYPE-B NONORTHOGONAL CABLE NETWORKS OF THE SAME SIDE LENGTH

Table 2

Natural frequencies (cps) for Type A non-orthogonal cable networks

Mode	2x3 Net			3x4 Net			4x5 Net			[illegible] Net		
	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]
1	57.88	60.59	57.86	58.43	–	53.4	53.66	–	[illegible]	[illegible]	[illegible]	[illegible]
2	82.46	90.01	82.437	86.48	–	86.44	88.41	–	88.36	[illegible]	[illegible]	[illegible]
3	91.17	102.10	91.138	93.45	–	93.42	94.02	–	93.99	94.18	96.95	94.14
4	98.54	112.03	98.505	109.98	–	109.94	113.69	–	[illegible]	115.29	120.38	115.25

Table 3

Natural frequencies (cps) for Type B non-orthogonal cable networks

Mode	2x3 Net			3x5 Net			4x7 Net			5x9 Net		
	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]	FETM	FEM [7]	TMM [21]
1	53.99	57.34	54.58	55.31	57.31	55.73	55.56	–	55.97	55.79	–	56.12
2	65.65	71.20	65.04	69.25	73.13	69.97	70.37	–	71.22	71.04	–	71.67
3	79.60	92.38	79.13	84.89	91.68	84.53	88.13	–	88.08	89.01	–	89.26
4	91.29	108.75	92.05	97.48	109.24	96.50	98.09	–	99.81	101.21	–	100.91

.	Mass density of the material of beam	: 7.86×10^{3} Kg/m^{3}
.	Shear modulus of the material of beam	: 89.0×10^{9} N/m^{2}
7.	Polar moment of inertia of beams lying in X and Y direction	: 1.040579×10^{-6} m^{4}
8.	Polar moment of inertia of equivalent circular section of beams lying in X and Y direction	: 0.3288034×10^{-6} m^{4}

4.6 RESULTS FOR ORTHOGONAL BEAM GRILLAGE WITH CLAMPED ENDS

4.6.1 BENDING VIBRATIONS ONLY

First four natural frequencies of the orthogonal beam grillage 2x2, 3x3, 4x4, 5x5 and 10x10, Figure 14, are calculated for the data given in Section 4.5 and are given in Table 4. These are compared with those obtained by FEM [18] and ~~plate~~ plate analogy of beam grillage [18].

Comparison of the results shows that in this case the results obtained by FETM method are quite close to those obtained by FEM and plate analogy. As discussed in the case of cable networks, the results obtained by FETM are exact within the computational errors. The present method has an advantage over FEM and the plate analogy because of its simplified formulation.

4.6.2 COMBINED BENDING-TORSIONAL VIBRATIONS

First four natural frequencies for the clamped orthogonal beam grillage 2x2, 3x3, 4x4, 5x5 and 10x10, Figure 14, are calculated for the data given in Section 4.5 and are given in Table 5.

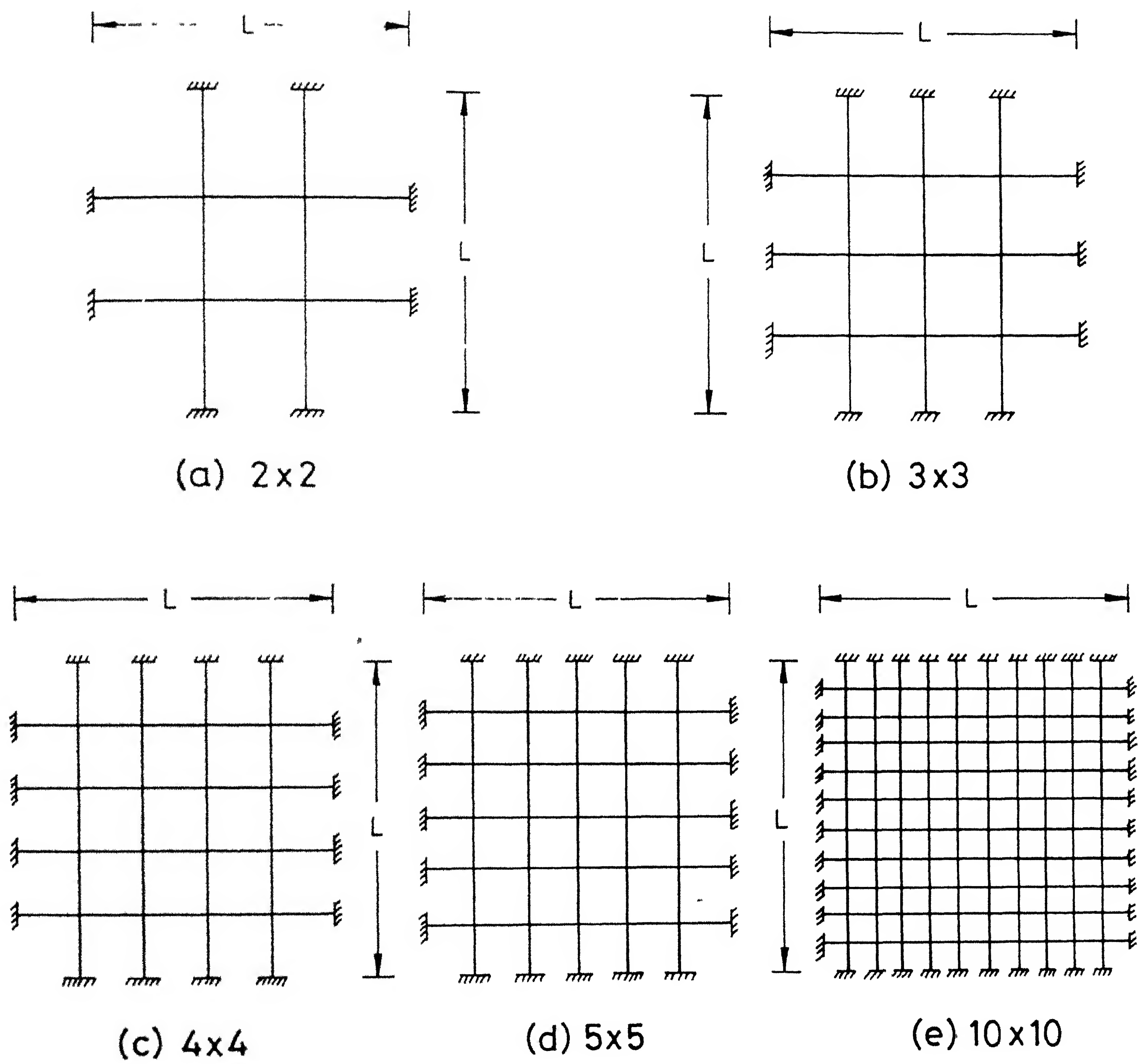


Fig. 14 Different orthogonal beam grillages with clamped edge of the same side length.

Table 4

Natural frequencies (cps) for the orthogonal beam grillage with clamped ends in bending vibrations only

Mode	2x2 grid			3x3 grid			4x4 grid			5x5 grid			10x10 grid
	FETM	FEM [18]	ANALOGY [18]	FETM	FEM [18]	ANALOGY [18]	FETM	FEM [18]	ANALOGY [18]	FETM	FEM [18]	ANALOGY [18]	FETM
A_{11}	83.36	83.41	82.8	83.12	83.18	82.8	83.07	83.11	82.8	83.05	83.09	82.8	83.039
A_{12}	160.51	162.04	160.8	170.09	170.89	170.0	171.59	171.98	170.2	171.95	172.18	171.1	172.2
A_{21}	162.23	162.04	160.8	170.09	170.89	170.0	171.59	171.98	170.2	172.09	172.18	171.1	172.2
A_{22}	233.62	233.55	225.2	230.31	231.10	225.2	229.42	229.90	225.2	229.13	229.43	226.1	228.91

Table 5

Natural frequencies for the orthogonal beam grillage with clamped ends in combined bending torsional vibrations

Mode	FETM (present method)				
	2x2 grid	3x3 grid	4x4 grid	5x5 grid	10x10 grid
A_{11}	91.21	93.84	95.12	95.36	97.2
A_{12}	168.38	184.99	190.13	192.34	196.4
A_{21}	168.38	184.99	190.13	192.54	196.4
A_{22}	233.18	260.99	273.01	279.94	292.4

Sharma [18] in the plate analogy and in the FEM assumed the beams to be torsionless i.e. the torsional stiffness of the beam elements was considered to be negligible and zero which leads to bending vibration only. When the torsional stiffness (G) of the element is considered, the grillage becomes more rigid as compared to that with negligible torsional stiffness and hence ~~are~~ one expects a higher value for the natural frequencies. Comparison of the data given in Tables 4 and 5 validates this justification.

4.7 RESULTS FOR ORTHOGONAL BEAM GRILLAGE WITH SIMPLY SUPPORTED ENDS

In this section, results for bending vibrations of orthogonal beam grillage with simply supported ends are presented. The natural frequencies for 2x2, 3x3 and 4x4 are given in Table 6 and compared with those of FEM [18] and plate analogy [18]. The results obtained are quite close to FEM and plate analogy. As discussed in the last section the present results are from the exact formulation and within the limit of computational errors.

4.8 MODE SHAPES

The mode shapes for the above mentioned cases ~~has~~ have been computed and some of these are shown in Figure 15 to Figure 17.

Table 6

Natural frequencies (cps) for the orthogonal beam grillage with simply supported ends in bending vibrations only

a. 2x2 grid

Mode	FETM (present method)	FEM [18]	Plate analogy [18]
A_{11}	36.63	36.67	36.64
A_{12}	105.6	106.34	106.8
A_{21}	105.6	106.34	106.8
A_{22}	146.525	148.31	146.6

b. 3x3 grid

Mode	FETM (present method)	FEM [18]	Plate analogy [18]
A_{11}	36.63	36.65	36.64
A_{12}	106.52	106.79	106.8
A_{21}	106.52	106.87	106.8
A_{22}	146.525	147.18	146.6

c. 4x4 grid

Mode	FETM (present method)	FEM [18]	Plate analogy [18]
A_{11}	36.63	36.65	36.64
A_{12}	106.70	106.71	106.8
A_{21}	106.70	106.83	106.8
A_{22}	146.525	144.67	146.6

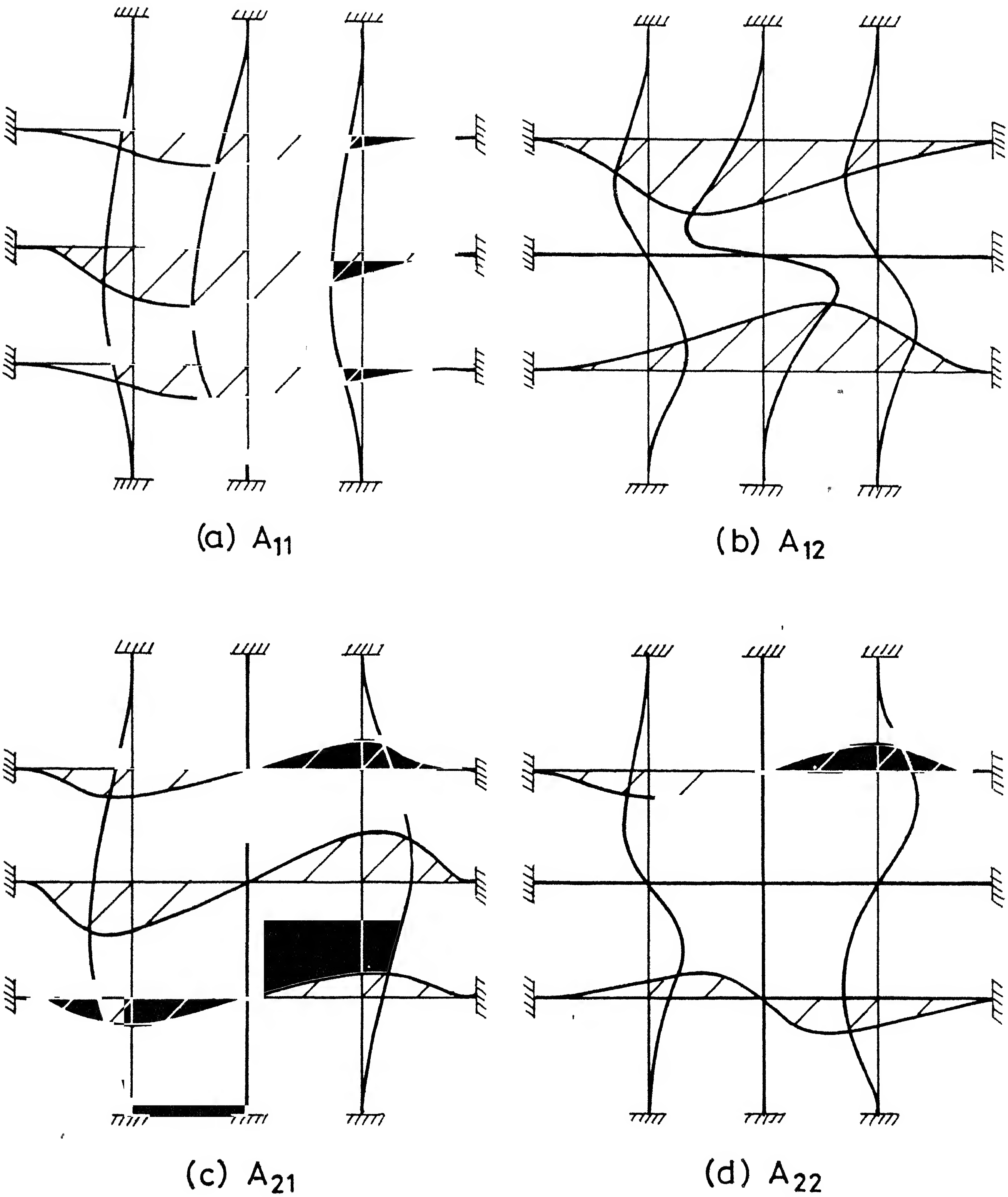


Fig. 15 First four mode shapes for 3x3 clamped beam grillage in bending vibrations only.

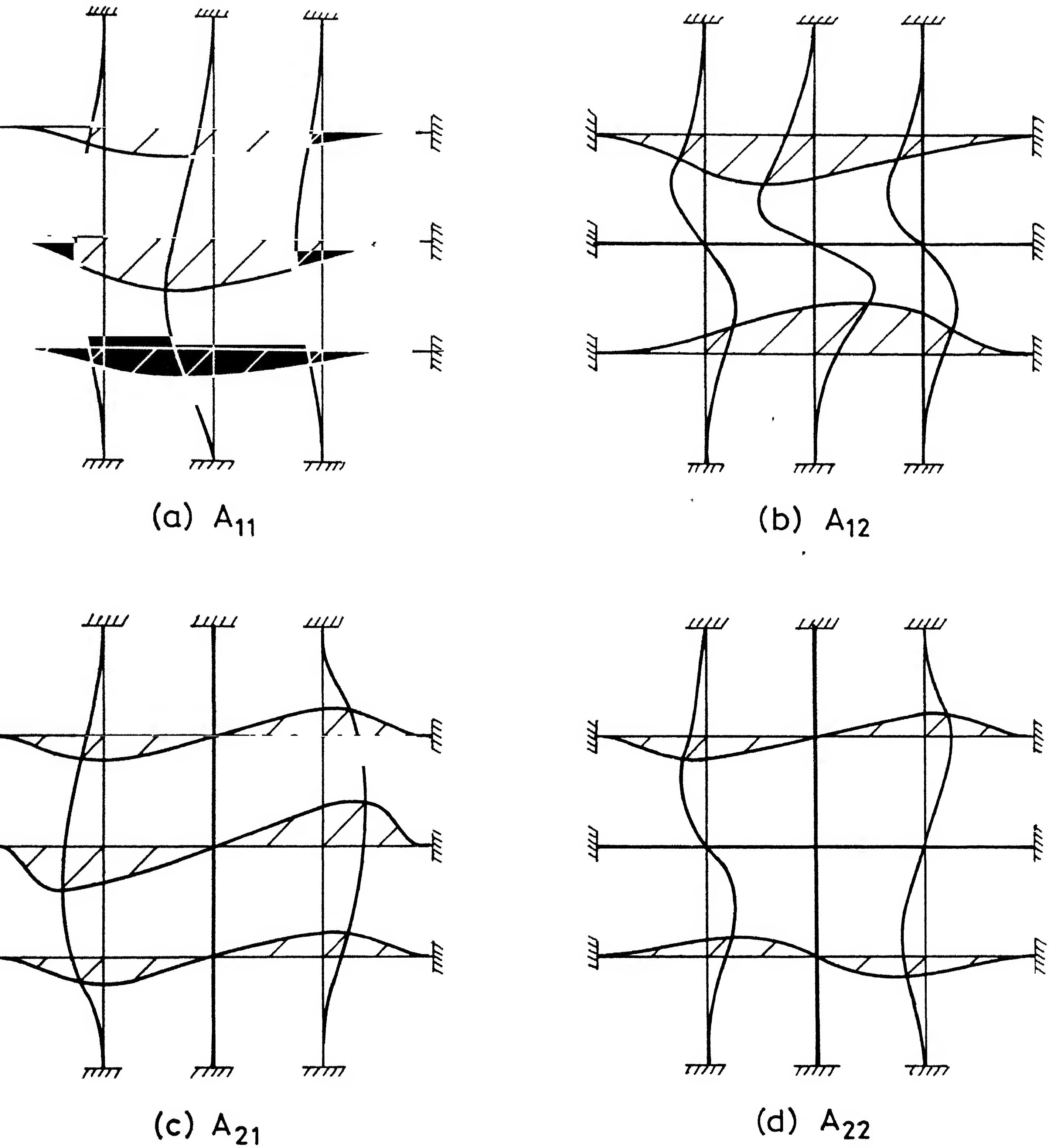


Fig. 16 First four mode shapes for 3x3 clamped beam grillage in combined bending torsional vibrations.

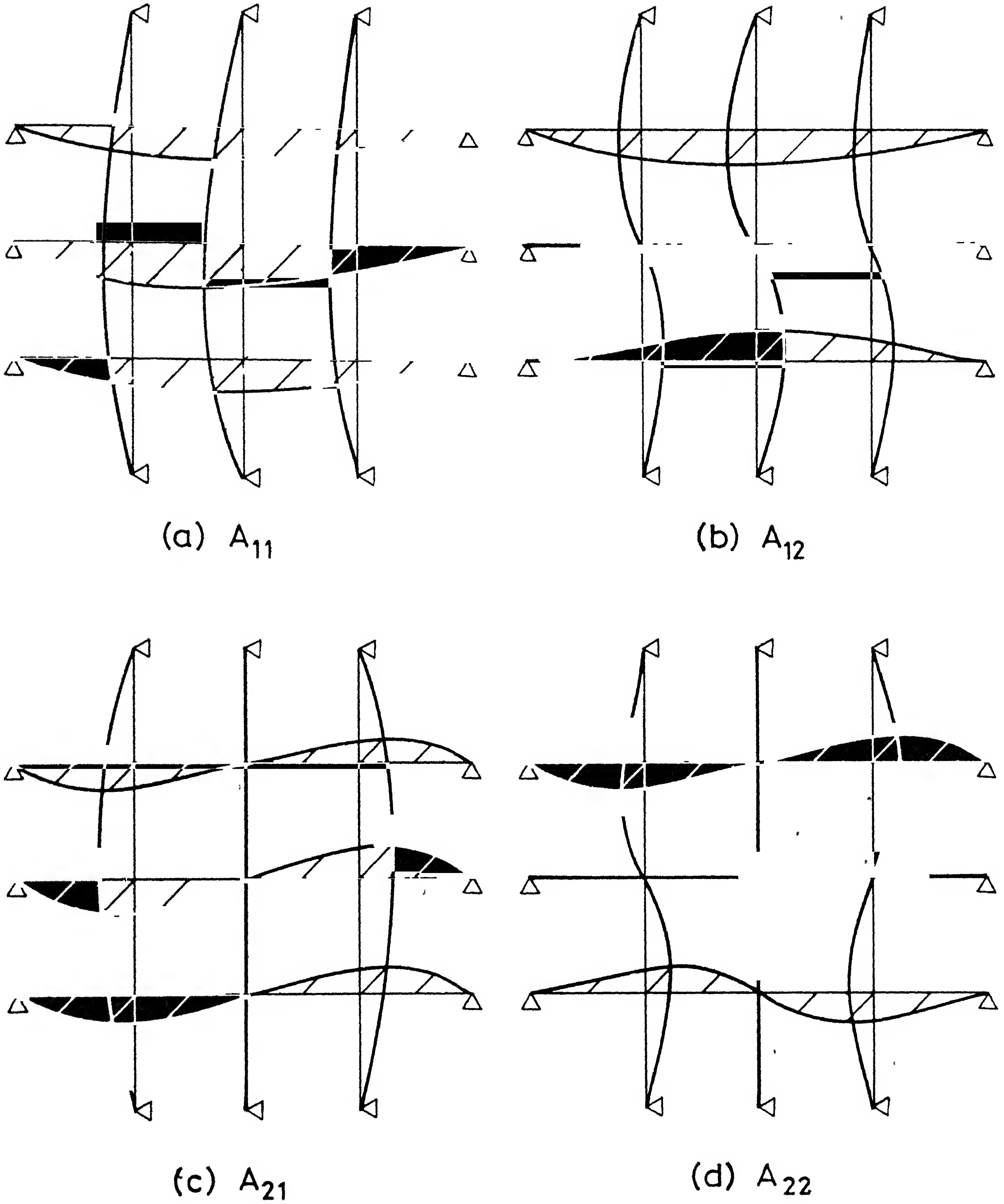


Fig. 17 First four mode shapes for 3x3 simply supported beam grillage in bending vibrations only.

CHAPTER 5

CONCLUSIONS

In the present work, vibration analysis of various cable networks and beam grillages have been carried out with the help of FETM method.

The natural frequencies corresponding to first four modes have been obtained for cable networks, orthogonal and non-orthogonal. Also the natural frequencies and mode shapes for beam grillage (a) in bending vibrations and (b) in combined bending-torsional vibrations have been obtained. Comparison of the results with those reported in literature shows that the results are comparable. Further, the formulation of the problem is obtained from the exact solution of its elements.

In the vibration analysis of cable networks by FETM method the size of the TM is equal to twice the number of cables lying in one direction for orthogonal cable networks and non-orthogonal cable networks Type A, whereas in the case of non-orthogonal cable networks Type B, the size is less than twice the number of cables lying in both the direction by two. But in FEM the matrix size is dependent on the number of elements being considered. Therefore the matrix size in the present method is far lower than the FEM but is equal to the matrix size in TMM. In the present method, the analysis is quite simple as compared to TMM. So, the simplicity of analysis is the advantage of present method over TMM.

In vibration analysis of beam grillage by the present method the size of TM is also small as compared to the FEM.

Based on the present work it can be concluded that FETM method is quite powerful and economical tool for the vibrational analysis of systems which have repeated nature.

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